Stochastic modelling of plasma reflection during keyhole arc welding

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Abstract
Keyhole arc welding (KAW), including the keyhole double-sided arc welding process being developed at the University of Kentucky and keyhole plasma arc welding, can achieve much deeper narrower penetration than all other arc welding processes. If it could be controlled such that the heat input and weld pool are minimized while at the same time the desired full penetration is guaranteed, it could become an effective yet affordable technology to improve productivity in welding thick materials. However, the key in developing such a controlled KAW technology is the sensor which can detect the evolution of the keyhole. Preliminary study shows that the plasma reflection could lead to a practical yet accurate sensor. In this study, the dynamic behaviour of the plasma reflection is described using the reflection arc angle (RAA). It is found that the RAA series can be considered an autoregressive moving-average (ARMA) process. The orders of the ARMA model are determined using auto-correlation and partial auto-correlation functions. The parameters of the ARMA are recursively estimated using the extended least squares algorithm. It is found that the recursive estimates of the model parameters change as the state of the keyhole changes. A discriminator has been proposed to determine the state of the keyhole based on the recursive estimates of the model parameters.

Keywords: plasma, ARMA, stochastic process, arc welding

1. Introduction

The keyhole plasma arc welding (PAW) process achieves much deeper penetration than all other existing arc welding processes. It uses a special torch with a constraining orifice [1]. Electrons emitted from the tungsten electrode flow through the ionized plasma gas and form a highly constrained plasma jet. This plasma jet melts the workpiece and can displace the molten metal to form a keyhole or deep narrow cavity [2]. In this way, the plasma jet can heat the workpiece through the thickness in a similar manner to the laser welding process. This change in the heating mechanism gives keyhole PAW strong penetration capability. Similarly, a keyhole can also be achieved during double-sided arc welding (DSAW), a novel process being developed at the University of Kentucky [3, 4], to further improve its already-enhanced penetration capability. At its current development stage, the keyhole DSAW is capable of penetrating in a single pass a workpiece up to 12.7 mm (1/2 inch) thick [5], which normally requires up to six passes to arc weld otherwise [6].

Unlike laser welding, where the keyhole is primarily established by the vaporization of the metal, the vaporization in keyhole arc welding (KAW), keyhole PAW or keyhole DSAW is much reduced. The keyhole is primarily established by the force of the high speed plasma jet on the molten metal. As a result, the weld pool is still larger or wider than that in laser welding. If the process can be controlled such that weld pool is minimized but the desired full penetration still achieved, KAW may become a primary method to weld thick materials.

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Keyhole arc welding

Figure 1. Behaviours of plasma reflection. (a) Non-penetrated keyhole. (b) Penetrated keyhole.

One method which appears capable of reducing the weld pool is to use a large current to establish the keyhole and then use a small current to avoid burn-through after the keyhole is established. It is evident that a successful implementation of this type of controlled KAW relies on reliable detection of the establishment of the keyhole. In the past, the light of the plasma efflux from the keyhole \[7\] and the spectral lines of hydrogen and argon \[8\] have been monitored to determine the establishment of a penetrated keyhole. However, to apply the proposed control technology, a more direct method is needed to determine the state of the keyhole.

Figure 1 demonstrates the behaviours of the plasma arc and its reflection during KAW. When the keyhole is not fully penetrated, the plasma arc has to be reflected from the cavity, which is referred to as non-penetrated keyhole. However, after the keyhole is fully established, the plasma jet will exit at the bottom of the penetrated keyhole. The amount of reflected plasma, if any, will be significantly reduced. Hence, it is possible to reliably monitor the establishment of the penetrated keyhole based on the behaviour of the reflected plasma.

To explore the possibility of monitoring the establishment of the penetrated keyhole based on the behaviour of the reflected plasma, a high speed image system was used to observe the plasma arc and its reflection \[9\]. The angle of the reflected plasma, referred to as reflection arc angle (RAA), was extracted. It was found that the development of the keyhole has three states or periods \[9\], the stable non-penetrated keyhole period, the instable transition period and the stable penetrated keyhole period. During the stable non-penetrated period, the RAA fluctuates around a small degree with small amplitudes. Once the development enters into the instable transition period, the RAA fluctuates around a larger degree with larger amplitudes. After the stable penetrated keyhole state is established, the RAA increases and is stabilized at an even larger degree.

Although the RAA exhibits a fundamental relationship with the state of the keyhole development, the determination of the state based on the RAA appears not to be straightforward because of the complexity of the dynamic behaviour of the RAA. In this study, the RAA will be modelled as a stochastic process and the recursive estimate of the model parameters will be used to determine the state of the keyhole.

2. ARIMA models for RAA series

Figure 2 shows two types of image. While the plasma arc and its reflection can be observed in both images, the efflux plasma can be observed only when the keyhole is fully penetrated.

Figure 3 demonstrates the proposed method which is used to extract the RAA in this study. In the proposed method, the middle vertical axis of the main plasma arc, denoted as aa, is first identified by detecting the highest and lowest point in the image. Then the median points referred to as effective points 1, 2, \ldots, 5 of the vertical lines \(l_1, \ldots, l_5\) are located. By utilizing the least squares method and treating the effective points as the observation points \[10\], the slope and intercept of the axis of the reflection arc can be obtained and used to generate the reflection arc angle \(\theta\). In particular, the regressive model for the least squares method is \[10\]

\[
\bar{Y} = \Phi \tilde{\theta}
\]

where

\[
\bar{Y} = [y(1), \ldots, y(5)]^T \quad \tilde{\theta} = [b, k]^T
\]

\[
\Phi = \begin{bmatrix}
1 & x(1) \\
\vdots & \vdots \\
1 & x(5)
\end{bmatrix}
\]

1965
Figure 3. Determination of reflection arc angle.

and \( k \) and \( b \) are the slope and intercept respectively for the line equation of the axis of the reflected plasma. The least squares estimate of the parameter vector \( \vec{\theta} \) is \[ \hat{\vec{\theta}} = \left( \Phi^T \Phi \right)^{-1} \Phi^T \vec{y}. \] (2)

Hence, the estimated RAA \( \vartheta \) is

\[ \vartheta = \tan^{-1}(k). \] (3)

Consequently, a virtual reflection arc line can be drawn using the slope \( k \) and intercept \( b \) in the two-dimensional image as shown in figure 3.

Figure 4 shows a binarized image series obtained from two cases, one for welding a 4.5 mm thick workpiece, and another for welding a 6.5 mm thick workpiece. In both cases, the keyhole welding process changed from the initial non-penetrated keyhole mode to the penetrated keyhole mode. Using the algorithm proposed above, the RAA can be obtained from the images. Figure 5 depicts the RAA series for the two image series given in figure 4.

As can be seen in the image series in figure 4(a), the first appearance of the efflux plasma is in image frame 49. However, it appears that the reflected plasma only settles down after frame 62. In the case of the 6.5 mm thick work-piece as shown in figure 4(b), the efflux plasma first occurs at the 113th frame, i.e. the 11th image in row 7. However, the efflux plasma remains always present only after the 169th frame, i.e. the 16th image in row 10. A previous analysis [9] showed that the development of the keyhole has three states or periods: the stable non-penetrated keyhole period, during which the non-penetrated keyhole develops, the instable transition period, during which the penetrated keyhole can be closed or a non-penetrated keyhole can become fully penetrated by slight disturbances, and the stable penetrated keyhole period, in which the penetrated keyhole maintains. The RAA exhibits different dynamic behaviours in different states. In the stable non-penetrated keyhole state, the non-penetrated keyhole grows and develops gradually; the geometry of the keyhole which determines the RAA fluctuates. As a result, the RAA fluctuates, but with small amplitudes around a small angle. During the transition period, the geometry of the keyhole is subject to significant fluctuation. The RAA oscillates at large amplitudes. After the keyhole is settled into the stable penetrated keyhole state, the geometry of the keyhole maintains nearly unchanged. The RAA thus varies with very small amplitudes around a large reflection angle.

Despite the characteristics of the RAA in different states, the RAA series in figure 5 looks too stochastic to determine the state. No explicit function into which the data set fits appears. For example, the mean and the variance of the RAA for the data set in figure 5(a) are 2.4532 and 0.0942, respectively. Careful observation shows that the mean and variance also vary with time. Hence, there is a possibility that the process might
be non-stationary [11]. The histogram for the RAA shown in figure 6 suggests some random distribution for the data set. Therefore, the data set should be considered a stochastic process.

In general, a non-stationary stochastic process can be described using an auto-regressive integrated moving-average (ARIMA) model [11]. Denote the time series to be modelled as \( \{z_t\} \). In this study, the time series to be modelled is the RAA series. The general equation for an ARIMA is

\[
wt = \phi_1wt_{t-1} + \cdots + \phi_p wt_{t-p} + \theta_1 e_{t-1} + \cdots + \theta_q e_{t-q}
\]

where \( wt = \nabla^dz_t \) is the \( d \)th difference of the time series \( \{z_t\} \) to be modelled, \( \nabla \) is the backward difference operator, \( \phi_i (i = 1, \ldots, p) \) and \( \theta_j (j = 1, \ldots, q) \) are the model parameters and \( e_1, \ldots, e_{t-q} \) are series of white noise with zero mean. Here, \( \nabla = 1 - B \), where \( B \) is the backward shift operator such that \( z_{t-1} = Bz_t \). Therefore, \( \nabla z_t = (1 - B)z_t = z_t - z_{t-1} \), \( \nabla^dz_t = (1 - B)^dz_t = wt \). Equation (4) can also be written as

\[
\phi(B)wt = \theta(B)e_t
\]

where

\[
\begin{align*}
\phi(B) &= 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p \\
\theta(B) &= 1 + \theta_1 B + \theta_2 B^2 + \cdots + \theta_q B^q.
\end{align*}
\]

Equations (5) and (6) give a general form for an ARIMA \((p, d, q)\) model where \( p, d, q \) are the model orders. The ARIMA \((p, d, q)\) model can describe both stationary \((d = 0)\) and non-stationary \((d > 0)\) time series.

The relationship between \( wt \) and \( z_t \) can also be described by \( z_t = (\nabla^d)^{-1}wt = (\nabla^{-1})^dwt \), where \( \nabla^{-1} = (1 - B)^{-1} \) is

Figure 5. Series of the RAA under different thickness. (a) 4.5 mm thickness; (b) 6.5 mm thickness
the summation operator defined by
\[ \nabla^{-1} w_t = \sum_{j=0}^{\infty} w_{t-j} = w_t + w_{t-1} + w_{t-2} + \ldots \] (7)

Hence, it is meaningful to include ‘integrated’ in naming the model.

3. Identification of ARIMA model structure

Accurate determination of the orders \((p, d, q)\) requires accurate description of the underlying physical process. For the RAA series, accurate determination of the model structure purely based on mathematical derivation requires very complicated analysis and modelling of the process physics, which are not available yet. It should be pointed out numerous approaches have been proposed to select the order of an ARMA model. For example, the AIC selects orders based on the concept of entropy [12]. The approach proposed by Fuchs is based on the rank determination of estimated covariance matrices [13]. Ribeiro and Moura determine the order by minimizing a functional \(d\) that measures the mismatch of the assumed model to the data [14, 15]. This functional is evaluated from the estimated reflection coefficient sequence associated with the process. In [16] and [17], an instrumental variable based and a Wald statistic based approach are proposed. In [18], one of the authors of the present paper proposed a minimum spectrum distance based criterion. However, despite the effectiveness of these methods, this application study will use graphical methods because they are relatively easy to implement and yet often lead to acceptable estimates of model models [11].

There are two established graphical techniques, auto-correlation function (ACF) combined with partial auto-correlation function (PACF) and canonical correlation. In this study, the ACF and PACF method will be implemented.

3.1. Identification of difference order by ACF

Determination of \(d\) is a crucial step in the identification of the model structure for an ARIMA process. It is known [11], for a stationary mixed auto-regressive moving average process of order \((p, 0, q)\) which can be represented by \(\phi(B)z_t = \theta(B)e_t\),\n
that the ACFs, denoted by \(\rho_k (k = 0, 1, \ldots)\), satisfy the difference equation
\[ \phi(B)\rho_k = 0. \] (8)
In order to describe the principle of the ACF method for determining $d$, let us first solve for $\rho_k$. Denote $\phi(B) = \prod_{i=1}^{p} (1 - G_i B)$ where $G_i (i = 1, 2, \ldots, p)$ is a root of $\phi(B)$. Assume distinct roots. The solution for (8) is

$$\rho_k = A_1 G_k^1 + A_2 G_k^2 + \cdots + A_p G_k^p \quad k > q - p \quad (9)$$

where $A_1, A_2, \ldots, A_p$ are constants determined by the initial values of data series.

Now let us describe the principle of the ACF method for determining $d$. Assume that $d = 0$, i.e. $z_t$ is an ARMA process described by $\phi(B) z_t = \theta(B) e_t$. Then the amplitude of all the roots of $\phi(B)$ must be greater than 1 [11], or all the roots must lie outside the unit circle. The roots of $\phi(B)$ are

$$B = 1 / G_i \quad i = 1, 2, \ldots, p. \quad (10)$$

This implies that $G_1, G_2, \ldots, G_p$ must lie inside the unit circle for $z_t$ to be a stationary ARMA process $\phi(B) z_t = \theta(B) e_t$. Hence, from (9), for a stationary ARMA process, the $\rho_k$ must die out as $k$ increases. Otherwise, the process is not stationary. Therefore, a tendency for the ACF not to die out quickly is an indication that a root close to unity may exist. It turns out logically that a failure of the ACF to die out rapidly might suggest that the underlying stochastic process $z_t$ is non-stationary. $z_t$ will need to be differentiated as many times as necessary until $w_t = \nabla^d z_t$ becomes stationary.

A satisfactory estimate of the ACF has been suggested by one statistician in [19]:

$$r_k = c_k / c_0 \quad (11)$$

where

$$c_k = \frac{1}{N} \sum_{t=1}^{N-k} (z_t - \bar{z})(z_{t+k} - \bar{z}) \quad k = 0, 1, 2, \ldots, K$$

and $K$ is the number of the ACF to be computed and estimated, $N$ is the number of samples of the time series, $\bar{z}$ is the mean of the data series and $c_k$ is the estimate of the auto-covariance $y_k$.

Table 1 gives $z_t, \nabla z_t, \nabla^2 z_t$ for the RAA series in figure 5(a). The estimated ACFs $r_k^{(0)}$ (for $z_t$), $r_k^{(1)}$ (for $\nabla z_t$) and $r_k^{(2)}$ (for $\nabla^2 z_t$) are shown in table 2. Figure 7 plots the estimated ACF for $z_t, \nabla z_t$ and $\nabla^2 z_t$.

As can be observed from figure 7, ACF decreases after $k = 1$ fairly significantly, and ACFs of $z_t, \nabla z_t$ decay exponentially combined with some sine waves. This suggests that an ARMA or ARIMA $(p, 0, q)$ model may be sufficient to describe the RAA. However, the quality of the model needs to include the further criterion partial auto-correlation function (PACF) to be verified.

### 3.2. Identification of ARMA process

In this sub-section, orders $p$ and $q$ for the AR and MA operators will be determined by studying ACF $\rho_k$ and PACF $\phi_k$. The characteristic of an AR($p$) process, $\phi_k = 0$ for all $k > p$, can be used to characterize the $p$th-order AR process, where

$$\phi_k = \text{corr}[z_t - \hat{z}_t, z_{t-k} - \hat{z}_{t-k}] \quad (12)$$

where $\hat{z}_t$s are the least squares regression estimations of $z_t$ based on $z_{t-1}, \ldots, z_{t-k}$, and $\hat{z}_{t-k}$s are least squares regression estimations of $z_{t-k}$ based on $z_{t-k+1}, \ldots, z_{t-1}$. Hence, $\phi_k$
measures the correlation between \( z_t \) and \( z_{t-k} \) after adjusting for \( \hat{z}_t \) and \( \hat{z}_{t-k} \). \( \phi_{kk} \) is accordingly called the partial auto-correlation function or PACF.

\( \phi_{kk} \) can be obtained as the solution of the Yule–Walker equation [20]. That is, given the estimated ACF \( \hat{\phi}_{pj} \), one has \( \phi_{pj} = \hat{\phi}_{pj} \). The parameters \( \phi_{pj} \) can be estimated using the following recursive equations [21]:

\[
\hat{\phi}_{p+1,1} = \hat{\phi}_{p1} - \hat{\phi}_{p1}\hat{\phi}_{p1} \quad j = 1, 2, \ldots, p \quad (13)
\]

\[
\hat{\phi}_{p+1,1} = r_{p+1} = \sum_{j=1}^{p} \hat{\phi}_{pj} r_{p+1-j} \left( 1 - \sum_{j=1}^{p} \hat{\phi}_{pj} r_{j} \right)^{-1} \quad (14)
\]

The initial values are calculated using equation (12).

With the calculated \( \phi_{kk} \), the graphic feature of the ACF combined with PACF can be used to evaluate the orders \( p \) and \( q \). In brief, whereas the auto-correlation function of an auto-regressive process of order \( p \) cuts off, its partial auto-correlation function has a cutoff after lag \( p \). Conversely, the auto-correlation function of a moving average process of order \( q \) has a cutoff after lag \( q \), while its partial auto-correlation tails off. If both the auto-correlation and partial auto-correlation tail off, a mixed ARMA process is suggested. Furthermore, the auto-correlation function for a mixed process, containing a \( p \)th-order auto-regressive component and a \( q \)th-order moving average component, is a mixture of exponential and damped sine waves after the first \( p + q \) lags. All the properties of the ACF and PACF for five common processes are summarized in table 3, while table 4 shows the PACFs \( \phi_{00}, \phi_{11}, \phi_{22} \) for \( z_t, \nabla z_t, \nabla^2 z_t \). Figure 8 are their graphic presentations.

It can be observed from figure 8(a) that the PACF is dominated by a mixture of exponential and damped sine wave. This suggests a mixture model with \( (2, 0, 1) \).

### 4. Recursive estimation of ARMA model parameters

The analysis in the last section suggests that the RAA be described as a mixed ARMA \((2, 1)\):

\[
(1 - \phi_1 B - \phi_2 B^2) z_t = (1 + \theta_1 B) e_t. \quad (15)
\]

For convenience, use \( z(t) \) and \( e(t) \) to denote \( z_t \) and \( e_t \). Equation (15) can be rewritten as

\[
z(t) = \phi_1 z(t-1) + \phi_2 z(t-2) + \theta_1 e(t-1) + e(t). \quad (16)
\]

Equation (16) can be considered as a stochastic version of the regression model. The parameters \( \phi_1, \theta_1 \) can be recursively estimated by using the extended recursive least squares algorithm [22].

The regression model is

\[
z(t) = [z(t-1) \quad z(t-2) \quad e(t-1)] \begin{bmatrix} \phi_1 \\ \phi_2 \\ \theta_1 \end{bmatrix} + e(t) \quad (17)
\]
where \( e(t - 1) \) can be approximated by the predicted error
\[
\hat{e}(t) = (z(t) - \hat{z}(t))
\]
\[
\hat{e}(t) = (z(t) - \hat{z}(t))
\]
where \( \hat{\theta}(t) = [\hat{\phi}_1(t - 1), \hat{\phi}_2(t - 1), \hat{\theta}_1(t - 1)]^T \) is the recursive estimate of the parameter vector at the previous instant. Hence, the recursive equations are
\[
\begin{align*}
\hat{\theta}(t) &= \hat{\theta}(t - 1) + p(t)\psi(t - 1)\hat{e}(t) \\
p(t) &= \frac{1}{\lambda + \psi^T(t - 1)p(t - 1)\psi(t - 1)}
\end{align*}
\]
where
\[
\hat{\theta}(t) = [\hat{\phi}_1(t - 1), \hat{\phi}_2(t - 1), \hat{\theta}_1(t - 1)]^T
\]
\[
\hat{\theta}(t) = [\hat{\phi}_1(t - 1), \hat{\phi}_2(t - 1), \hat{\theta}_1(t - 1)]^T
\]
and \( \lambda \) is the forgetting factor.

5. Analysis using recursive estimates of parameters

Figure 9 shows the curves of the estimated parameters under a thickness of 4.5 mm. It can be seen that at the beginning of the recursive estimation period, i.e. prior to frame 21, the estimates fluctuate. This is typical for recursive algorithms because of the large covariance gain. After this beginning period, the estimates become smooth. This indicates that the process is in a stable state. However, at approximately frame 50, the recursive estimates begin to change. This indicates a change in the development state of the keyhole. At approximately frame 60, the recursive estimates approach another stable state. As can be seen in figure 4(a), the keyhole is first fully penetrated at frame 49. Also, the behaviour of the reflected plasma reaches the stable penetrated keyhole state at approximately frame 61. As can be seen from the recursive estimates, the recursive estimates start to change after frame 50 and become converged again at approximately frame 56. Hence, it is possible to use the changes in the recursive estimates of the ARMA models to determine the change in the state of the development of the keyhole.

Figure 10 shows the recursive estimates of the RAA during welding of a 6.5 mm thick work-piece. Figure 4(b) shows that the keyhole is first fully penetrated at frame 113 and is finally settled at the new stable penetrated keyhole state at frame 169. Figure 10 shows that the recursive parameters converged before frame 113. However, only \( \theta_1 \) starts to change substantially right after frame 113. The other two parameters \( \phi_1, \phi_2 \) start to substantially change after another 20 frames. In this case, although \( \phi_1, \phi_2 \) did not change right after the first full penetration of the keyhole, the change in \( \theta_1 \) was drastic. Such a pattern in parameters can still suggest a very different ARMA process after frame 113. Hence, the transition period can still be predicted based on the recursive parameters. Further, approximately after frame 170, all the parameters converge again. This indicates that the process enters into another stable state. Of course, this state is that of the stable penetrated keyhole.

It appears that the recursive estimates of the ARMA(2, 1) model can be used to determine the change in the state of the keyhole process. However, there are three parameters in an ARMA(2, 1) model. To determine the change in the state, three parameters need to be used as the inputs of a decision or discriminator function.

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### Table 3. Behaviour of ACF for \( d \)th difference of ARIMA \((p, d, q)\) process [11].

<table>
<thead>
<tr>
<th>Order</th>
<th>((1, d, 0))</th>
<th>((0, d, 1))</th>
<th>((2, d, 0))</th>
<th>((0, d, 2))</th>
<th>((1, d, 1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Behaviour of (\rho_0)</td>
<td>Decays exponentially</td>
<td>Only (\rho_1) nonzero</td>
<td>Mixture of damped sine wave</td>
<td>Only (\rho_1) and (\rho_2) nonzero</td>
<td>Decays exponentially from first lag</td>
</tr>
<tr>
<td>Behaviour of (\phi_{xx})</td>
<td>Only (\phi_{11}) nonzero</td>
<td>Exponentially dominates decay</td>
<td>Only (\phi_{11}) and (\phi_{22}) nonzero</td>
<td>Dominated by mixture of damped sine wave</td>
<td>Dominated by exponential decay from first lag</td>
</tr>
</tbody>
</table>

### Table 4. PACFs \(\phi_{00}, \phi_{11}, \phi_{22}\) for \(z_t, \nabla z_t, \nabla^2 z_t\).

<table>
<thead>
<tr>
<th>lag</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_{00})</td>
<td>1–10</td>
<td>0.5505</td>
<td>-0.2187</td>
<td>-0.0608</td>
<td>0.0247</td>
<td>0.0439</td>
<td>0.0522</td>
<td>-0.0053</td>
<td>0.019</td>
<td>-0.0615</td>
</tr>
<tr>
<td></td>
<td>11–20</td>
<td>0.0649</td>
<td>-0.0494</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\phi_{11})</td>
<td>1–10</td>
<td>0.1301</td>
<td>-0.0881</td>
<td>-0.0222</td>
<td>-0.1262</td>
<td>0.0324</td>
<td>-0.0133</td>
<td>0.0278</td>
<td>0.2054</td>
<td>-0.055</td>
</tr>
<tr>
<td></td>
<td>11–20</td>
<td>-0.0535</td>
<td>-0.1026</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\phi_{22})</td>
<td>1–10</td>
<td>-0.2194</td>
<td>-0.0816</td>
<td>0.0301</td>
<td>-0.1051</td>
<td>0.0338</td>
<td>-0.0457</td>
<td>-0.1135</td>
<td>0.0533</td>
<td>0.1639</td>
</tr>
<tr>
<td></td>
<td>11–20</td>
<td>0.0886</td>
<td>-0.2337</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
Figure 9. Estimated parameters of data series under 4.5 mm thickness. (a) First estimated parameter, $\phi_1$; (b) second estimated parameter, $\phi_2$; (c) third estimated parameter, $\theta_1$. 

1972
Figure 10. Estimated parameters of data series under a 6.5 mm thickness. (a) First estimated parameter, $\phi_1$; (b) second estimated parameter, $\phi_2$; (c) third estimated parameter, $\theta_1$. 
To propose a simple yet effective discriminator or decision function which determines the state of the keyhole process, let us consider the keyhole process as a system with a white noise as the input and the RAA as the output. The identified ARMA(2, 1) model is thus the impulse transfer function of the system. Denote the impulse transfer function as $H(z)$. Then,

$$H(z) = \frac{1 + \theta_1 z^{-1}}{1 - \phi_1 z^{-1} - \phi_2 z^{-2}}. \quad (21)$$

The frequency response of the system is [23]

$$H(e^{j\omega T}) = \frac{1 + \theta_1 e^{-j\omega T}}{1 - \phi_1 e^{-j\omega T} - \phi_2 e^{-j2\omega T}} \quad (22)$$

where $\omega$ is the frequency and $T$ is the sampling period, which is 0.001 s in this study. Hence, the recursive estimates of the parameters can be used to compute the system’s frequency response at frequencies of interest.

The RAA series shown in figure 5 suggest that during the stable non-penetrated keyhole state the RAA fluctuates with smaller amplitudes than it does during the transition period. The system which produces the RAA from the white noise should thus have higher high-frequency gains during the stable non-penetrated keyhole than it has during the transition period. Further, in the stable penetrated keyhole state, the fluctuation of the RAA is nearly negligible. The high-frequency gains must be the lowest while the low-frequency gains must be the highest among the three states.

Based on the above discussion, a discriminator function is proposed:

$$f = \left| H(e^{j\omega_H T}) \right| / \left| H(e^{j\omega_L T}) \right| \quad (23)$$

where $\omega_H$ and $\omega_L$ are the selected high and low frequency, respectively. Using $\omega_H = 2\pi \times 100$ rad $s^{-1}$ and $\omega_L = 2\pi \times 10$ rad $s^{-1}$, the recursive estimates in figures 9 and 10 are used to compute the frequency responses at both the low and high frequency of selection and the discriminator function. The results are illustrated in figure 11(a) and figure 11(b) for the cases of a 4.5 mm thick work-piece and a 6.5 mm thick work-piece, respectively.

As can be seen in figure 11(a) for the thinner (4.5 mm thick) plate, the change in the state of the keyhole development can be determined from the discriminator function. In fact, the discriminator function decreases rapidly after frame 50 and settles after frame 56. This indicates that at frame 50 the keyhole process changes its state from the non-penetrated keyhole to the transition period. However, in this case (thinner plate), the duration of the transition period is very brief, and the process quickly enters the stable penetrated keyhole state. Because of such a quick transition, one can consider that the process enters the stable penetrated keyhole state directly from the stable non-penetrated keyhole state. The anticipated ‘middle’ values of the discriminator function associated with the transition state are thus not observed.

Similarly, the change in the state of the keyhole development can be determined from the discriminator function for the 6.5 mm thicker plate as can be seen in figure 11(b). The images in figure 4(b) shows that the keyhole is fully penetrated at frame 113 and finally settles at the new stable penetrated keyhole state at frame 169. However, prior to frame 113, the reflection arc has started to increase its fluctuation amplitude. Such an increase in the fluctuation amplitude implies that the keyhole is about to fully penetrate. The keyhole process is thus about to change its state into the transition state. As can be seen in figure 11(b), the discriminator function sharply decreases prior to frame 98. This sudden decrease suggests that the keyhole has started its change from the stable non-penetrated keyhole to the transition period at frame 98. Hence, the discriminator function may provide a possible prediction, which is needed for advanced real-time control of the keyhole process, for the occurrence of the transition period in advance. On the other hand, as can be seen in figure 4(b), the keyhole opens and closes a few times before it finally settles at the stable penetrated keyhole state at frame 169. This type of fluctuation suggests a mixture of the transition state and the non-penetrated keyhole. Hence, the discriminator function may exhibit high values again during this period before the process finally settles at the stable penetrated keyhole state at frame 169. As
can be seen, the discriminator function does exhibit high values again. However, such recurrence of high values only lasts a brief period and the discriminator function returns to the low values rapidly. Approximately at frame 140, the discriminator function decreases to a level which predicts the stable penetrated keyhole state.

As can be seen that for the thicker plate, the discriminator function is capable of predicting the change of the state in advance. This is primarily due to the fact that the change of the state takes a long time to complete in the thicker plate. However, for the thinner plate, the change of the state completes much more quickly. Hence, for the thinner plate, the prediction function of the discriminator function is not pronounced although the change of the state can be determined based on the discriminator function. However, for specific applications, further studies must be conducted to learn specific dynamic behaviours of the reflection arc, which may vary with the thickness, material etc, in order to find the thresholds for automated prediction and determination of the change of the keyhole state.

6. Conclusions

This study used a high speed image processing system to acquire images during KAW. The RAA was extracted from the image to describe the dynamic behaviours of the reflected plasma. It was found that the RAA series is a stochastic process which can be described using an ARMA model.

The parameters of the ARMA model were recursively estimated. It was found that the recursive estimates can converge if the state of the keyhole remains unchanged. However, if the state of the keyhole changes, at least some of the recursive estimates of the parameters substantially change accordingly. This implies that the parameters of the ARMA change with the state of the keyhole.

The ARMA model can be considered the impulse transfer function of the ARMA system which produces the RAA process as the output from a white noise input. The frequency response of the ARMA system has different characteristics in different states. Based on the characteristics in the frequency response, a discriminator function can be formed to simplify the decision-making in detecting the state of the keyhole and improve the detection accuracy and robustness. For the thicker plate, for which the change of the state takes a certain time to complete, the discriminator function can also predict the change of the state in advance.

It should be pointed out that this study is focused on the development of the methodology. For specific applications, further studies must be performed to determine the thresholds needed for automated prediction and determination of the change of the keyhole state.

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