Given two Gaussian r.v.s, \( x_1, x_2 \), with zero mean and covariance
\[
C_{xx} = \begin{bmatrix} 1 & -3/4 \\ -3/4 & 1 \end{bmatrix}
\]  
(1)

We want to find a transformation
\[
y = A x
\]  
(2)

where \( y = [y_1, y_2]^T \), \( x = [x_1, x_2]^T \) and \( A \) is 2x2.

**Step 1: Find eigenvalues of \( C_{xx} \).**

The eigenvalues are found by setting the determinant of \( C_{xx} - \lambda I \) equal to 0:
\[
|C_{xx} - \lambda I| = 0
\]  
(3)

For the values in Eq. (1) we have
\[
(1 - \lambda)^2 - 9/16 = 0 = \lambda^2 - 2\lambda + 7/16
\]  
(4)

\( \lambda_1, \lambda_2 = 7/4, 1/4 \)  
(5)

**Step 2: Find eigenvectors of \( C_{xx} \).**

We know the relationship of eigenvalues and eigenvectors is
\[
C_{xx} e_i = \lambda_i e_i
\]  
(6)

and in matrix form
\[
C_{xx} E = E \Lambda
\]  
(7)

where the columns of \( E \) are the eigenvectors and the diagonal of \( \Lambda \) contains the eigenvalues. Also note that for orthonormal case, \( E^T = E^{-1} \)

The eigenvalue/eigenvector relationship is
\[
(C_{xx} - \lambda I)e_i = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} c_{11} - \lambda_i & c_{12} \\ c_{21} & c_{22} - \lambda_i \end{bmatrix} \begin{bmatrix} e_{i,1} \\ e_{i,2} \end{bmatrix}
\]  
(8)

The Eq. (8) will give two sets of equations but for reasons we will not go into, we will only use the first equation such that
\[
(1 - \lambda_i) e_{i,j} - (3/4) e_{2,j} = 0 \tag{9}
\]

Let \( i = 1 \) so Eq. (9) is
\[
(1 - 7/4) e_{1,j} = (-3/4) e_{1,j} = (3/4) e_{2,j} \tag{10}
\]

Let \( i = 2 \) so Eq. (9) is
\[
(1 - 1/4) e_{1,j} = (3/4) e_{1,j} = (3/4) e_{2,j} \tag{11}
\]

We see that for one eigenvector \( e_{1,1} = -e_{2,1} \) and for the other eigenvector \( e_{1,2} = e_{2,2} \). The inner product of an eigenvector with itself is unity for an orthonormal eigenvector so if we let the element values have values
\[
E = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \tag{12}
\]

**Step 3: Diagonalize \( C_{yy} \)**

Letting \( A = E^T \) such that
\[
y_i = e_i^T x = e_{i,j} x_1 + e_{2,j} x_2 \tag{13}
\]

we can diagonalize \( C_{yy} \) such that
\[
C_{yy} = E \{ y y^T \} = A E \{ x x^T \} A^T = A C_{xx} A^T = E^T C_{xx} E = E^T E \Lambda = \Lambda \tag{14}
\]

**Step 4: Whitening \( C_{yy} \)**

From Eq. (14), we can see that if we weighted \( A \) by \( \Lambda^{-1/2} \) we would get an identity matrix for \( C_{yy} \). The algebra for this would be based on letting \( A = \Lambda^{-1/2} E^T \) such that
\[
C_{yy} = E \{ y y^T \} = A E \{ x x^T \} A^T = \Lambda^{-1/2} E^T C_{xx} E \Lambda^{-1/2} = \Lambda^{-1/2} E^T E \Lambda \Lambda^{-1/2} \tag{15}
\]

and so because the eigenvalue matrix is diagonal we can use algebra to complete the whitening as
\[
C_{yy} = \Lambda^{-1/2} \Lambda \Lambda^{-1/2} = \Lambda^{-1} \Lambda = I \tag{16}
\]