Quadrature Modulation and Demodulation

Quadrature detection

The binary detector has its origins in narrow band deterministic quadrature detection.

\[ \cos \left(2\pi f_c t + \theta \right) \]

\[ g_c(t) = \cos(2\pi f_c t) \cos(2\pi f_c t + \theta) = \frac{1}{2} \left[ \cos \theta + \cos(4\pi f_c t + \theta) \right] \]

\[ g_s(t) = \sin(2\pi f_c t) \cos(2\pi f_c t + \theta) = \frac{1}{2} \left[ -\sin \theta + \sin(4\pi f_c t + \theta) \right] \]
Detect presence of signal

\[ \cos \theta \rightarrow \frac{1}{11} \rightarrow 1 = \cos^2 \theta + \sin^2 \theta \]

\[ \sin \theta \rightarrow \frac{1}{11} \rightarrow \text{quadrature detector} \]

\[ M \text{ symbol quadrature Modulation/Remodulation} \]

Assume we have two signals \( a(t) \) and \( b(t) \)
where each signal has \( M \) levels.
Let \( T \) be the symbol interval
We combine $a(t)$ and $b(t)$ using quadrature modulation, i.e., they share the same modulation spectrum.
where $H_L(f)$ is a band limiting filter.

$$S(t) = a(t) A_c \cos 2\pi f_c t + b(t) A_c \sin 2\pi f_c t$$

$$S(f) = A_c \left( A'(f) * \frac{S(f-f_c) + S(f+f_c)}{2} \right) + A_c \left( B'(f) * \frac{S(f-f_c) - S(f+f_c)}{2} \right)$$

$$S(f) = \frac{A_c A(f)}{2} \quad \text{Re} \quad -f_c \quad f_c \quad f$$

$$\frac{A_c |A(f)|^2}{2} \quad \text{Im} \quad f_c$$
Channel Noise

\[ \tilde{r}(t) = s(t) + \tilde{\omega}(t) \]

\[ \tilde{\omega}(t) \text{ AWGN} \]

Quadrature Demodulation

\[ H(f) \text{ may be a LP filter} \]

\[ H(f) \text{ is an "integrate and dump" matched filter} \]

The entire "mixer/IF filter" is replaced by a single matched filter.
\[ \tilde{y}_c(t) = \tilde{r}(t) \cos 2\pi f_c + \tilde{\omega}_c(t) \cos 2\pi f_c \]

\[ = s(t) \cos 2\pi f_c + \tilde{\omega}_c(t) \cos 2\pi f_c \]

\[ = a'(t) A_c \cos^2 2\pi f_c + b'(t) A_c \sin 2\pi f_c \cos 2\pi f_c + \tilde{\omega}_c(t) \cos 2\pi f_c \]

\[ \text{O from LPF} \]

\[ \tilde{\omega}_c(t) \cos 2\pi f_c \]

\[ \text{LPF} \]

\[ \tilde{y}_s(t) = a'(t) \frac{A_c}{2} + \tilde{\omega}_c(t) \]

\[ \text{Likewise for the other leg} \]

\[ \tilde{y}_s(t) = a'(t) A_c \sin \frac{\pi f_c}{2} \cos \frac{\pi f_c}{2} + b'(t) A_c \sin \frac{\pi f_c}{2} \cos \frac{\pi f_c}{2} + \tilde{\omega}_s(t) \text{ LPF} \]

\[ \text{after LPF} \]

\[ \tilde{y}_s(t) = b'(t) \frac{A_c}{2} + \tilde{\omega}_s(t) \]}
Map $\tilde{y}_c(t)$ and $\tilde{y}_s(t)$ to a signal space.

at time $t = t_0$.

$M = 8$ possible symbols

circular constellation & MPE decision boundary

We use 8 different values of $a(t)$

01 if we used a rectangular

constellation $m = 4^2$
M-ary likelihood

For binary ML we used a ratio as the detector.
If we have M cases, then there are common ways
to configure the detector.

1. Given $f(x|H_i)$ for $i=0,1,\ldots,(M-1)$

   Form $m-1$ ratios $L_{\text{rat}}(x) = \frac{f(x|H_{i+1})}{f(x|H_0)}$ for $i=1,2,\ldots,(M-1)$

   Look for maximum ratio

2. Look for max $\sum_i f(x|H_i)^3$