We know
\[ f_\tilde{z}(z) = \int_{-\infty}^{\infty} f_\tilde{X}(w) f_\tilde{Y}(z-w) \, dw = f_\tilde{X}(z) * f_\tilde{Y}(z) \]

We can successively apply this relationship to a sum of many i.i.d. r.v.'s and obtain what is known as the Central Limit Theorem. 

Theorem: Let $\tilde{X}_1, \tilde{X}_2, \ldots, \tilde{X}_n$ be i.i.d. r.v.'s with cdfs $F_1(x)$, $F_2(x)$, \ldots, $F_n(x)$.

We will assume $\mu_{\tilde{X}_k} = 0$ and $\text{Var} (\tilde{X}_k) = \sigma_k^2$. 

Let \( \tilde{Z}_n \triangleq \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \tilde{X}_i \).

\[
\lim_{n \to \infty} f_{\tilde{Z}_n}(z) = \frac{1}{\sqrt{2\pi} \sigma_z} \exp \left( -\frac{(z - \mu_z)^2}{2\sigma_z^2} \right)
\]

\[
E \tilde{Z} \tilde{Z}_n^3 = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} E \tilde{Z}_i \tilde{X}_i^3 = 0
\]

\[
\text{Var} \left( \tilde{Z}_n \right) = E \left( \tilde{Z}_n^2 \right) = \frac{1}{n} \sum_{i=1}^{n} E \tilde{X}_i^2
\]

\[
= \frac{1}{n} \sum_{i=1}^{n} \sigma_i^2
\]

\[
\text{SUMMATIONS OF ANY IND. RVs APPROACH A GAUSSIAN RV}
\]
Example: Central Limit Theorem (CLT)

\[ \tilde{Z} = \tilde{X}_1 + \tilde{X}_2 + \cdots + \tilde{X}_N \]

and \( \tilde{X}_i \) are all independent

\[ \lim_{N \to \infty} \tilde{Z} \sim N(\mu_Z, \sigma_Z^2) \]

Example: \( \tilde{Z} = \tilde{X}_1 + \tilde{X}_2 + \tilde{X}_3 \) where \( \tilde{X}_i \) are iid

let \( \tilde{\omega} = \tilde{X}_1 + \tilde{X}_2 \)

\[ f_{\tilde{\omega}}(\omega) = \begin{cases} 1 & 0 < \omega < 1 \\ 0 & \text{otherwise} \end{cases} \]

\[ \tilde{Z} = \tilde{\omega} + \tilde{X}_3 \]

\[ f_{\tilde{Z}}(z) = \begin{cases} 1 & 0 < z < 1 \\ \text{more complex like} \end{cases} \]