Let $N = 512$:

1. Uniform pseudo-random numbers. Generate 6 random vectors, each with a different seed. The vectors are all $N^2 \times 1$ where each element is uniformly distributed between 0 and 1. Each element is independent from the others. Mathematically refer to the vectors as:

$$u_1, u_2, u_3, u_4, u_5, u_6,$$ (1)

2. Prove the parametric transformation equations for converting from a uniform distribution to a gaussian distribution are correct.

$$g_{[2i+1]} = \sqrt{-2 \ln u_{[2n+1]}} \cos 2\pi u_{[2n+2]}$$ (2)

$$g_{[2i+2]} = \sqrt{-2 \ln u_{[2n+1]}} \sin 2\pi u_{[2n+2]}$$ (3)

where $n = 0, 1, 2 \ldots (N^2/2 - 1)$

3. Generate six $N^2 \times 1$ gaussian random vectors from the associated vectors in part (1). Use the transformation developed in (2). Generate them with a 0 mean and unity variance and store as you did in (1). Refer to them as

$$g_1, g_2, g_3, g_4, g_5, g_6,$$ (4)

4. Linear combinations of r.v.s. Generate five $N^2 \times 1$ vectors such that

$$s_1 = u_1 + u_2$$

$$s_2 = u_1 + u_2 + u_3$$

$$s_3 = u_1 + u_2 + u_3 + u_4$$

$$s_4 = u_1 + u_2 + u_3 + u_4 + u_5$$

$$s_5 = u_1 + u_2 + u_3 + u_4 + u_5 + u_6$$ (5)

5. Change $g_i$ from lexicographical form to 2D matrix $G_i$. Put the first $N$ elements in $g_i$ to the first row of $G_i$, the $N - 2N$ elements in $g_i$ to the second row of $G_i$ and so on. The size of $g_i$ is $N^2 \times 1$ and the size of $G_i$ is $N \times N$.

6. Letting $N = 128$: Use the 12 training images from Project 1-S: Supplemental “SYNTHESIS OF DETERMINISTIC SYSTEM” and add zero mean white Gaussian Noise to them such that the Signal to Noise Ratio is 0 dB. You can use randn() to generate the noise for the 6 target and 6 clutter images. Add them to the images and subplot 12 noisy images.
7. Generate two more 1-D vectors, a pseudo-random binary (i.e., bipolar) sequence and pseudo-random intensity sequence such that:

\[ b_{\text{binary}}[n] = \begin{cases} 
 1 & \text{for } u_1[n] \geq 0.5, \\
 -1 & \text{for } u_1[n] < 0.5.
\end{cases} \]  

(6)

and

\[ s_{\text{intensity}}[n] = \left( g_1[n] \right)^2 \]  

(7)

where \( n = 1, 2, \ldots N^2 \). We will use these to conduct some signal processing experiments.