We can numerically approximate mean and variance for a finite number of sample values as for $N$ samples $X_i$.

\[
\text{mean} = \mu_s = \frac{1}{N} \sum_{i=1}^{N} X_i
\]

\[
\text{Variance} = \sigma_s^2 = \frac{1}{(N-1)} \sum_{i=1}^{N} (X_i - \mu_s)^2
\]

If we know the statistics (i.e., pdf or cdf) of a r.v., we can find the mean and variance exactly.
Let $X \sim f_x(x)$ and $Y = g(X)$

and then

$E[g(X)] = \int_{\mathbb{R}} g(x) f_x(x) \, dx$  \hspace{1cm} \text{Ensemble mean}$

$m = E[X^3] = \int_{\mathbb{R}} x^3 f_x(x) \, dx$  \hspace{1cm} \text{The moments of a linear operator are not we can rewrite as}$

$\mu_2 = E[(X-\mu)^2] = \int_{\mathbb{R}} (x-\mu)^2 f_x(x) \, dx$  \hspace{1cm} \text{second moment}$

and

$\mu_2 = \int_{\mathbb{R}} (x-\mu)^2 f_x(x) \, dx + \int_{\mathbb{R}} 2\mu x f_x(x) \, dx + \int_{\mathbb{R}} \mu^2 f_x(x) \, dx$  \hspace{1cm} \text{second moment}$

$\mu_2 = \frac{3}{2}$
\begin{align*}
\sigma^2 &= E(x)^2 - \mu^2 \\
&= E(3x^2) - \mu^2 \\
&= 3E(x^2) - \mu^2 \\
&= 3\int_0^1 x^2 f_X(x) \, dx - \mu^2 \\
&= 3\int_0^1 x^2 (x) \, dx - \mu^2 \\
&= 3\int_0^1 x^2 \, dx - \mu^2 \\
&= 3\left[ \frac{x^3}{3} \right]_0^1 - \mu^2 \\
&= 1 - \mu^2 \\
\end{align*}
Consider a discrete valued r.v. $X$.

Let $P_i = \text{Prob}(X = X_i)$ for $i = 1, 2, \ldots, N$

then

$$1 = \sum_{i=1}^{N} P_i$$

is a probability mass function (p.m.f.)

The mean value is

$$E(X) = \sum_{i=1}^{N} x_i \cdot P_i$$

If $P_i = \frac{1}{N}$ for $i = 1, 2, \ldots, N$ then

$$E(X) = \sum_{i=1}^{N} \frac{x_i}{N} = \sum_{i=1}^{N} x_i \cdot \frac{1}{N}$$

$$\mu = E(X)^3 = \left(\sum_{i=1}^{N} x_i \cdot \frac{1}{N}\right)^3$$

Similar to the convention.
The variance is

\[ \sigma^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2 \]