**Problem P4.2:** The cylindrical coordinate robot on a factory's assembly line is shown in a top view. The 50-N force acts on a work piece being held at the end of the robot's arm. Express the 50-N force as a vector in terms of unit vectors $\mathbf{i}$ and $\mathbf{j}$ that are aligned with the $x$- and $y$-axes.

**Figure P4.2**

![Diagram of the robot arm with force vector](image)

**Approach:**
Find the angle $\theta$ between the force and the $x$-axis. Calculate rectangular components using $F=50$ N and Equation (4.2): $F_x = F\cos\theta$ and $F_y = F\sin\theta$

**Solution:**
The 50-N force makes a 20° angle with a line perpendicular to the robot arm, and a 50° angle relative to vertical, since the arm itself is inclined by 30°. The force is therefore rotated 140° from the positive $x$-axis.

$\theta = 140^\circ$

$F_x = (50 \text{ N})\cos 140^\circ = -38.3 \text{ N}$

$F_y = (50 \text{ N})\sin 140^\circ = 32.1 \text{ N}$

$\mathbf{F} = -38.3\mathbf{i} + 32.1\mathbf{j} \text{ N}$

**Discussion:**
The components of the force are almost equal because the force is angled close to a 45° slope relative to the $x$-axis. If the force were to flatten out, the $\mathbf{i}$ component of the force would increase and the $\mathbf{j}$ component would decrease.
Problem P4.3: During the power stroke of an internal combustion engine, the 400 lb pressure force pushes the piston down its cylinder. Determine the components of that force in the directions along, and perpendicular to, the connecting rod \( AB \).

**Approach:**
Use the \( x\)-\( y \) coordinates attached to the connecting rod as shown. Find the rectangular components using Equation (4.2): \( F_x = F \cos \theta \) and \( F_y = F \sin \theta \) with \( F = 400 \) lb and \( \theta = 15^\circ \).

**Solution:**
The force components are:
\[
\begin{align*}
F_x &= (400 \text{ lb}) \cos 15^\circ = 386 \text{ lb} \\
F_y &= (400 \text{ lb}) \sin 15^\circ = 104 \text{ lb}
\end{align*}
\]
386 lb along the connecting rod
104 lb perpendicular to it

**Discussion:**
The force along the connecting rod is larger since the angle between the force and connecting road is smaller than the angle between the force and the perpendicular to the connecting road. As the piston moves to the left, the force along the connecting rod will decrease and the force perpendicular to it will increase.
Problem P4.4: A vector polygon for summing 2–kN and 1–kN forces is shown. Determine (a) the magnitude $R$ of the resultant by using the law of cosines and (b) its angle of action $\theta$ by using the law of sines.

![Figure P4.4](image)

Approach:
Apply the laws of cosines and sines from Appendix B:

- $c^2 = a^2 + b^2 - 2ab \cos c$
- $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

where $A$, $B$, and $C$ are interior angles, and $a$, $b$, and $c$ are side lengths. Angle $C = 90° - 55° = 35°$.

Solution:
(a) Magnitude of resultant from Equation (B.17):

$$R^2 = (7 \text{ kN})^2 + (2 \text{ kN})^2 - 2(7 \text{ kN})(2 \text{ kN})\cos 35°$$

$$R^2 = 50.49 + 4 - 28\cos 35°$$

$$R = 5.48 \text{ kN}$$

(b) Angle of action from Equation (B.16):

$$\frac{2 \text{ kN}}{\sin B} = \frac{5.48 \text{ kN}}{\sin 35°}$$

$$\sin B = 0.2092$$

$$B = 12.08°$$

$$A = 180° - 12.08° - 35° = 132.9°$$

$$\theta = 270° - A = 270° - 132.9° = 137.1°$$

Discussion:
The resultant force is less than the sum of the two forces because the vertical 2-kN force counteracts part of the vertical component of the 7-kN force.
Problem P4.6: The three tension rods are bolted to a gusset plate. Determine the magnitude and direction of their resultant. Use the (a) vector algebra and (b) vector polygon methods. Compare the answers from the two methods to verify the accuracy of your work.

Approach:
Using the unit vectors, combine the horizontal and vertical components by the vector algebra method using Equations (4.4)–(4.6). Apply Equation (4.7):
\[ R = \sqrt{R_x^2 + R_y^2} \text{ and } \theta = \tan^{-1} \left( \frac{R_y}{R_x} \right) \]
to find the resultant's magnitude and angle of action. Make a scale drawing of the vector polygon and measure the resultant's length and angle graphically.

Solution:
(a) Vector algebra approach
\[ F_1 = (500 \text{ lb}) (-\cos 20^\circ \mathbf{i} + \sin 20^\circ \mathbf{j}) = -469.8 \mathbf{i} + 171.0 \mathbf{j} \text{ lb} \]
\[ F_2 = (250 \text{ lb}) \mathbf{j} \]
\[ F_3 = (400 \text{ lb}) (\cos 45^\circ \mathbf{i} + \sin 45^\circ \mathbf{j}) = 282.8 \mathbf{i} + 282.8 \mathbf{j} \text{ lb} \]
\[ R = (-469.8 + 0 + 282.8) \mathbf{i} + (171.0 + 250 + 282.8) \mathbf{j} \text{ lb} \]
\[ = -187 \mathbf{i} + 703.8 \mathbf{j} \text{ lb} \]
\[ R = \sqrt{(-187 \text{ lb})^2 + (703.8 \text{ lb})^2} = 728.2 \text{ lb} \]
\[ \theta = \tan^{-1} \left( \frac{703.8 \text{ lb}}{-187 \text{ lb}} \right) = -75.12^\circ \]
The inverse tangent calculation would place the vector in the fourth quadrant, although it actually lies in the second quadrant since \( R_x < 0 \) and \( R_y > 0 \). Making this adjustment for the principal value of the angle, the proper angle, counterclockwise from \( \mathbf{i} \), is \( 180^\circ - 75.1^\circ = 104.9^\circ \).

\( 728 \text{ lb}, 104.9^\circ \text{ counterclockwise from } \mathbf{i} \)

(b) Vector polygon method
Add the three vectors graphically on a scale drawing using the head–to–tail rule. The magnitude and angle are measured from the drawing as \( 730 \text{ lb and } 105^\circ \), which confirms the calculation from part (a).
Discussion:
The answers from the two methods match, as expected. The resultant force is less than the sum of the forces because the forces are oriented in different directions. Also note that some of the horizontal components of the 400-lb and 500-lb forces offset each other. Increasing the 250-lb force would increase the \( j \) component of the resultant force, but would not impact the \( i \) component.