Circuits II
EE221
Unit 9
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Mutual Inductance, Energy in Magnetically Coupled Circuits, Analysis of Mutual Inductance Circuits
Mutual Inductance

Coils with current flowing in them emit magnetic energy that induce voltages in other coils in close proximity. The effects of the magnetic link between these circuits is called mutual inductance.

Show that current $i_1$ in first coil induces voltage $v_2$ in second coil of the form:

$$v_2 = M_{21} \frac{di_1}{dt}$$

where

$$M_{21} = N_2 \frac{d\phi_{12}}{di_1}$$

![Diagram showing mutual inductance between two coils](image-url)
**Mutual Inductance Dot Convention**

The geometry of the coil and flux paths are expressed in the dot convention below. This notation is important for applying circuit laws for analysis.
Dot Notation for KVL

If both currents are either entering or leaving the dot then the self and mutual induced voltage will have the same sign.

\[
\begin{align*}
V_1 &= I_1 L_1 s + I_2 M s \\
V_2 &= I_2 L_2 s + I_1 M s
\end{align*}
\]

If one current (loop direction) enters the dot and the other current leaves the dot, then the self and mutual induced voltage will have opposing signs.

\[
\begin{align*}
V_1 &= I_1 L_1 s - I_2 M s \\
V_2 &= I_2 L_2 s - I_1 M s
\end{align*}
\]
Equivalent Circuits Example

Convert the Mutual Inductance components to an equivalent circuit using current controlled voltage sources.

\[ V_1 = I_1 L_1 s + I_2 M s \]
\[ V_2 = I_2 L_2 s + I_1 M s \]
Examples

Find \( v_o(t) \), given \( v_i(t) = 2\cos(4t) \) \( V \).

Show \( v_o(t) = 0.4851\cos(4t-14^\circ) \) \( V \)
Examples

Find *Norton equivalent circuit wrt ab*

![Circuit Diagram]

Show $I_{th} = 0.69 \angle 6.4^\circ \, \text{A}$ \hspace{1cm} $Z_{th} = 19.52 \angle 87.1^\circ \, \Omega$
Examples

Find equivalent inductance wrt ab

Show \( L_{eq} = \frac{24}{7} \) H