Circuits II
EE221
Unit 5
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Passive Filters, low-Pass and Band-Pass filters
Filters

- A filter is a circuit/system designed with specifications for its amplitude and phase frequency response.

- Since capacitors and inductors result in frequency dependent impedance elements, it is possible to create circuits that filter out (or pass) signal energies over specified frequency regions.
Filter Example:

- Find the transfer function for the circuit below and sketch magnitude of the transfer function. Describe the filtering properties of this circuit with input $v_i(t)$ and output $v_o(t)$:

- Show that this filter can be described as a low-pass filter. (i.e. as $\omega \to \infty$, then $|\hat{H}(j\omega)| \to 0$, and as $\omega \to 0$, then $|\hat{H}(j\omega)| \to \text{Gain at DC} > 0$).
Capacitive and Inductive Elements:

- Consider the impedance magnitudes of the capacitor and inductor as a function of $\omega$:
  
  Capacitor: $|\hat{X}_C| = \frac{1}{|\omega C|}$  
  Inductor: $|\hat{X}_L| = |\omega L|$

- For higher frequencies the capacitor impedance approaches short circuit behavior while the inductor impedance approaches open circuit behavior.
- For lower frequencies the inductor impedance approaches short circuit behavior while the capacitor impedance approaches open circuit behavior.

- **Describe the capacitor impedance at DC.**
- **Describe the inductor impedance at DC.**
Basic Filter Types and Terminology

- The most common types of filters are listed below:
  - Low-pass
  - High-pass
  - Band-pass
  - Band-reject (or notch)

- **Filter Terms:**
  - **Passband** - range of input signal frequencies that pass through filter relatively unimpeded.
  - **Stopband** - range of input signal frequencies that are strongly attenuated (relative to the passband) in passing to the output.
  - **Cut-off frequency** - the end of the passband/stopband region. For the band-pass and band-reject filters there are 2 cut-off frequencies.
  - **3 dB cut-off (or half power) frequency** - the frequency where the magnitude of the transfer function is 3 dB down from its maximum value.
  - **Passive filter** - filter circuit without amplifier elements (no external power). The gain for passive filters is always less than or equal to 1.
  - **Active filter** - filter circuit using an amplifier element (i.e. an op amp). While the active filter requires power external to the input signal, it has greater flexibility for gain setting and superior buffering capabilities (i.e. filter properties do not change with the load).
Band-Stop Filter Design Example:

- For the band-reject filter given below, determine the circuit parameters so that the center (anti-resonant) frequency is 15 kHz, and the bandwidth 1.5 kHz. Determine the resulting gain at anti resonance, DC, and as $\omega \to \infty$.

![Diagram of band-stop filter with input $v_i(t)$, output $v_o(t)$, R, L, and C connected in series and parallel.]

- Show that

$$f_0 = \frac{1}{(2\pi)\sqrt{LC}} \quad \text{and} \quad B = \frac{1}{(2\pi)RC}$$

$G_{DC} = G_\infty = 1$

- In general, if a second order band-reject filter can be put in canonical form:

$$\hat{H}(s) = \frac{G(s^2 + \omega_0^2)}{s^2 + Bs + \omega_0^2}$$

where $G$ = gain at DC ($G_{DC}$) = gain as $\omega \to \infty$ ($G_\infty$), $B$ = 3 dB bandwidth, $\omega_0$ = center frequency. The gain at anti-resonance is 0.
Canonical TF Forms for Basic Filters:

- For the 2\textsuperscript{nd} order band-pass
  \[ \hat{H}(s) = \frac{G_o Bs}{s^2 + Bs + \omega_o^2} \]

- For the 2\textsuperscript{nd} order band-reject
  \[ \hat{H}(s) = \frac{G_o (s^2 + \omega_o^2)}{s^2 + Bs + \omega_o^2} \]

- For the 1\textsuperscript{st} order low-pass
  \[ \hat{H}(s) = \frac{G_{DC}}{\left(\frac{s}{\omega_c}\right) + 1} \]

- For the 1\textsuperscript{st} order high-pass
  \[ \hat{H}(s) = \frac{sG_\infty}{s + \omega_c} \]

where $G_{DC}$ is the gain at DC, $G_\infty$ is the gain at infinity, and $\omega_c$ is the 3 dB (half-power) cutoff frequency, and $\omega_o$ is the resonant/center frequency.