Circuit Laws

Ohm’s Law, Kirchhoff’s Law, Single loop circuits, Single node-pair circuits
Ohm’s Law

- The relationship between voltage and current through a material, characterized by resistance, is given by Ohm’s Law:
  \[ v(t) = R \times i(t) \]

- Resistive elements will always absorb power (i.e. convert electric energy to heat):
Units for Resistance/Conductance

- Resistance can be characterized in units of \textit{ohms}:\
  \[ R = 1\Omega = \frac{1V}{1A} \]

- Conductance can be characterized in units of \textit{siemens}:\
  \[ G = 1S = \frac{1A}{1V} \]
Examples

- Find the voltage across a 10Ω resistor when a current of 4 A is passing through it.

- Find the current in a .02S resistor when a voltage drop of 10 V occurs across it.

- Find the resistance of an element that exhibits a 14 V drop when 21 A pass through it.
Power in a Resistive Element

- Power absorbed by a resistor is given by:

\[ p(t) = v(t)i(t) \]

\[ p(t) = v(t) \frac{v(t)}{R} = \frac{v^2(t)}{R} = v^2(t)G \]

\[ p(t) = i(t)Ri(t) = i^2(t)R = \frac{i^2(t)}{G} \]
Open and Short Circuits

- If a resistance value goes to infinity, no current flows through it. This is referred to as an open circuit.

- If a resistance value goes to zero, no voltage drops across it. This is referred to as a short circuit.
Examples

- Solve for quantities (voltage, power, current, resistance) in resistive circuits with simple connections to independent and dependent sources.
Lumped-Parameter Circuit

- To represent the flow of electrical charge through an actual circuit, a zero-resistance connector is used to connect symbols denoting electrical properties of circuit parts.
Lumped-Parameter Circuit

Define and Identify Nodes, Branches, Loops

Node - connection between 2 or more circuit elements
Branch - circuit portion containing a single element
Loop - closed path containing no node more than once
**Kirchhoff’s Current Law (KCL)**

The sum of all currents entering a node (or any closed surface) equals zero

- Label each branch current and write a set of equations based on KCL
- Draw an arbitrary surface containing several nodes and write an equation based on KCL
**KCL Statements and Sign**

Sum of all currents leaving a node equals zero.
- Denote leaving as positive and entering as negative.

Sum of all currents entering a node equals zero.
- Denote entering as positive and leaving as negative.

Sum of all currents leaving a node equals sum of all current entering the node.
- Place all currents entering a node on one side of equation and all currents leaving the node on the other side.
Examples

- For circuits containing independent sources, dependent sources, and resistors, use KCL and Ohm’s Law to solve for unknown currents and voltages OR determine relations between quantities that cannot be resolved (i.e. when more unknowns than independent equations exist).
Kirchhoff’s Voltage Law (KVL)

- The sum of all voltages around any loop equals zero
- Label each branch voltage and write a set of equations based on KVL
KVL Statements and Sign

Sum of all voltage drops around a loop equals zero.
- Denote drops as positive and rises as negative.

Sum of all voltage rises around a loop equals zero.
- Denote rises as positive and drops as negative.

Sum of all voltage rises equals the sum of all voltage drops around a loop.
- Place all voltage rises on one side of equation and all voltage drops on the other side.
Voltage Labeling

\[ V_{R0} = V_{ab} = -V_{ba} \]

\[ V = V_{ac} = -V_{ca} \]

\[ V_o = V_R = V_{bc} = -V_{cb} \]
Examples

For circuits containing independent sources, dependent sources, and resistors, use KVL to solve for unknown voltages and currents or determine relation between quantities that cannot be resolved (i.e. when more unknowns than independent equations exist).
Single Loop and Node Circuits

- Solving for circuit quantities will involve the following steps:
  - Labeling the circuit
  - Deriving a set of equation from circuit
  - Solving the resulting equations
Find reduced expressions for all unknown voltages and currents \((V_1 = 5v, V_2 = -2v, R_1 = 3k\Omega, R_2 = 2k\Omega, R_3 = 5k\Omega)\):

\[ I = \frac{V_2}{R_1} \]

\[ V_1 = \frac{I R_2}{R_3} \]

Hint: Current in a single loop is the same through all elements, therefore use KVL.
Find reduced expressions all unknown voltages and currents ($I_1 = 1\, mA$, $R=4\, k\Omega$):

Hint: Voltage over a single node pair is the same over all elements, therefore use KCL.
For single loops with resistive elements and a voltage source, the following formula can be used to compute the voltage drop across any resistor:

\[ V_k = V_s \frac{R_k}{R_1 + R_2 + \cdots + R_N} \]
For single node pairs with resistive elements and a current source, the following formula can be used to compute the current in any resistor:

\[ I_k = I_s \frac{G_k}{G_1 + G_2 + \cdots + G_n} \]