Homework 1: (a) Use Eqn. (1.62) in (1.63) to derive (1.71a).  
(b) Use (1.71a) in (1.60) to derive (1.71b).

• Antennas: used for transmitting and receiving signals  
  Example: radar, communications

• Operation frequency: low, midium, high, very high, ultra high, microwave, …

• Construction: Printed, aperture, linear (wire), reflector, etc.

• Analysis and design:

Antenna's physical constructions  
(geometric parameters)  

Analysis  

Design  

Antenna's operation characteristics

• Major parameters of antennas:
  Input impedance
  Radiation pattern
  Polarization
  Efficiency
  Gain

• Other specifications:
  Size
  Weight
  Power capacity
  Reliability
  Shape
Antenna performance and operation frequency

1. **Electrically small antennas:**
   - Electrical size is much smaller than a wavelength
   - Very low directivity
   - Low input resistance and high input reactance
   - Low radiation efficiency
   - Example: small dipole, small loop
   - Application: Low frequency.

2. **Resonant Antennas:**
   - Electrical size is comparable to a wavelength.
   - Low to moderate gain
   - Real input impedance (small reflection)
   - Narrow bandwidth
   - Example: Half-wave dipole, single microstrip patch
   - Application: PCS communication

3. **Broadband antenna**
   - Real input impedance
   - Low to moderate gain
   - Gain, impedance remain at wide frequency band.
   - Example: Log-periodic array, sprial antenna.
   - Application: Multipurpose communication.

3. **Aperture antennas:**
   - Electric size much larger than wavelength
   - High gain
   - Moderate bandwidth
   - Example: Horn, Reflector antennas.
   - Application: Satellite signal receiving, Radar.
• **Typical Design cycles**

(1) Parameter specification

(2) Preliminary design

(3) Detailed design using theoretical equations, empiric formulas, and computer programs.

(4) If output parameters are satisfactory, done. Else go to (3) if more iterations are needed. Or re-specify parameter and go to (2).

Important: knowledge of antenna theory
   Working experience
   Knowledge in using computer programs

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**Example:**
Design a satellite TV signal receiving antenna that has directivity of more than 32dB at 12GHz. Preliminary design: reflector antenna, circular shape. Theoretical equation for directivity:

\[ D = \frac{4\pi A^2}{\lambda^2} = \frac{4\pi (\pi a^2)}{\lambda^2} \]

32dB ---- D=1585, 12GHz ---- \( \lambda = \frac{0.3}{12} = 0.083m \)

\[ a = \sqrt{\frac{D\lambda^2}{4\pi^2}} = \frac{\lambda}{2\pi} \sqrt{D} = \frac{0.083}{2 \times 3.14159} \sqrt{1585} = 0.526m \]
• Maxwell's equation in frequency domain

Time factor: $e^{j\omega t}$
Time domain field = Re\{Frequency domain field $X e^{j\omega t}$ \}

\[ \nabla \times \vec{E}(\vec{r}) = j\omega \mu \vec{H}(\vec{r}) - \vec{M} \]
\[ \nabla \times \vec{H}(\vec{r}) = -j\omega \varepsilon \vec{E}(\vec{r}) + \vec{J} \]
\[ \nabla \cdot \varepsilon \vec{E}(\vec{r}) = \rho(\vec{r}) \]
\[ \nabla \cdot \mu \vec{H}(\vec{r}) = 0 \]

$\vec{M}$: Equivalent magnetic current (does not physically exist)
$\vec{J}$: Electric current (applied or equivalent)

• Boundary condition: General

\[ \hat{n} \times (\vec{E}_2 - \vec{E}_1) = -\vec{M} \]
\[ \hat{n} \times (\vec{H}_2 - \vec{H}_1) = \vec{J} \]

• Boundary condition: conducting interface

$\vec{E}_{tan} = 0, \quad \vec{H}_{tan} = \vec{J}_s$
• Radiation solution

Vector potential: \( \vec{H} = \frac{1}{\mu} \nabla \times \vec{A} \)

Scalar potential: \( \Phi : \vec{E} + j\omega \vec{A} = -\nabla \Phi \)

Lorentz gauge: \( \nabla \cdot \vec{A} = -j\omega \varepsilon \mu \Phi \)

Equation for \( \vec{A} \):

\[ \nabla^2 \vec{A} + k^2 \vec{A} = -\mu \vec{J} \]

Solution:

\[ \vec{A}(\vec{r}) = \mu \int_s \frac{e^{-jkR}}{4\pi R} \vec{J}(\vec{r'})d\vec{r'} , \quad R = |\vec{r} - \vec{r'}| \]

\[ \vec{H} = \frac{1}{\mu} \nabla \times \vec{A} \]

\[ \vec{E} = \frac{1}{j\omega \varepsilon} \nabla \times \vec{H} , \quad \text{in source-free region.} \]

Key: The current distribution on the antenna surface.

• Ideal Dipole:

Infinitesimal element of current
Length \( \Delta z \ll \lambda \)
Current density: \( I = \text{constant} \)
Orientation: \( \hat{z} \)
Location: origin
Vector potential:

\[
\vec{A}(\vec{r}) = \mu \int_{-\Delta/2}^{\Delta/2} \frac{e^{-j\beta R}}{4\pi R} (\hat{z} I) \, dz'
\]

\[
= \hat{z} \frac{\mu I \Delta z e^{-j\beta R}}{4\pi r} \text{sinc} \left( \frac{1}{2} \beta \Delta z \cos \theta \right)
\]

Magnetic field:

\[
\vec{H}(\vec{r}) = \frac{1}{\mu} \nabla \times \vec{A} = \nabla \times \left( \hat{z} I \Delta z \frac{e^{-j\beta R}}{4\pi r} \right)
\]

Using a vector identity:

\[
\nabla \times (\phi \vec{B}) = \nabla \phi \times \vec{B} + \phi \nabla \times \vec{B} = 0
\]

\[
\vec{H} = \nabla \left( \Delta z I \frac{e^{-j\beta R}}{4\pi r} \right) \times \hat{z} = \frac{I \Delta z}{4\pi} \frac{\partial}{r} \left( \frac{e^{-j\beta R}}{r} \right) \times \hat{z}
\]

\[
= \frac{I \Delta z}{4\pi} j \beta \left(1 + \frac{1}{j \beta r}\right) \frac{e^{-j\beta R}}{r} \sin \theta \hat{\phi}
\]

\[
\vec{E} = \frac{I \Delta z}{4\pi} j \omega \mu \left[1 + \frac{1}{j \beta r} + \frac{1}{(j \beta r)^2}\right] \frac{e^{-j\beta R}}{r} \sin \theta \hat{\theta}
\]

\[
+ \frac{I \Delta z}{2\pi} j \omega \mu \left[1 + \frac{1}{j \beta r} + \frac{1}{(j \beta r)^2}\right] \frac{e^{-j\beta R}}{r} \cos \theta \hat{\phi}
\]
Three regions:

(i) Far-field region (radiation region): $\beta r >> 1$

$$\vec{E} = \frac{I\Delta z}{4\pi} j\omega \mu \frac{e^{-j\beta r}}{r} \sin \theta \hat{\theta}$$
$$\vec{H} = \frac{I\Delta z}{4\pi} j\beta \frac{e^{-j\beta r}}{r} \sin \theta \hat{\phi}$$

Important properties:

1. Field amplitude $\propto \frac{1}{r}$
2. Fields do not have $\hat{r}$ component
3. $\vec{E}, \vec{H}, \hat{r}$ form right-hand-rule and $\vec{E} = \eta \vec{H} \times \hat{r}, \quad E/H = \sqrt{\varepsilon/\mu} = \eta$

(ii) Intermediate region: $\beta r \approx 1$

(iii) Near-field region: $\beta r << 1$

$$\vec{E} = \frac{I\Delta z}{4\pi} j\omega \mu \frac{e^{-j\beta r}}{(j\beta)^2 r^3} \left(\hat{\theta} \sin \theta + 2\hat{r} \cos \theta\right)$$

** Angular pattern is the same as an electrostatic dipole.
• **Radiation pattern:**

Angular variation of electric field amplitude or phase at far-field region. It is usually normalized so that the maximum value is 1. In the above ideal dipole example, the radiation pattern is

\[ F(\theta, \phi) = \sin \theta \]

Graphical representation: often plot in polar-coordinate system.
Far-field calculation

Under far-field approximation, the radiation integrals to calculate radiate fields can be simplified.

The potential integral now become

$$A = \mu \int_s \frac{e^{-j\beta(r - \hat{r} \cdot \hat{r}')}}{4\pi r} J(\hat{r}')d\hat{r}' = \frac{\mu e^{-j\beta r}}{4\pi r} \int_s e^{j\beta\hat{r} \cdot \hat{r}'} J(\hat{r}')d\hat{r}'.$$

$$E = -j\omega A$$  (for $\hat{\theta}$ and $\hat{\phi}$ components)

That is: $E_\theta = -j\omega \hat{\theta} \cdot A$, $E_\phi = -j\omega \hat{\phi} \cdot A$

The radiation patterns:

$$\begin{bmatrix} F_\theta(\theta,\phi) \\ F_\phi(\theta,\phi) \end{bmatrix} = \frac{-j\omega \mu}{4\pi} \int_s e^{j\beta\hat{r} \cdot \hat{r}'} \begin{bmatrix} \hat{\theta} \cdot \hat{J} \\ \hat{\phi} \cdot \hat{J} \end{bmatrix} d\hat{r}'$$
• **Far-field distance** $r_{ff}$:

The distance where the phase error is $\pi/8 = 22.5^o$

$$R = |\vec{r} - \vec{r}'| = r - \hat{r} \cdot \vec{r}' + \frac{1}{2} \left( \frac{r'}{r} \right)^2 \left[ 1 - (\hat{r} \cdot \hat{r}')^2 \right] + \cdots$$

Phase error $\leq \frac{1}{2} \left( \frac{r'}{r} \right)^2 \beta$

If $D$ is the diameter of the antenna volume (defined as the diameter of the smallest sphere that encloses the antenna), then $r' \leq D/2$, hence

$$\frac{1}{2} \left( \frac{D/2}{r_{ff}} \right)^2 \frac{2\pi}{\lambda} = \frac{\pi}{8}, \quad r_{ff} = \frac{2D^2}{\lambda}$$

The distance calculated above is usually used as a minimum distance criteria for antenna measurement. For example, to measure an antenna of diameter 1m at frequency of 10GHz, the minimum distance (distance between transmission and receiving antennas) should be $2 \times 1^2 / 0.03 = 66.7m$
• Steps to obtain radiation field:

(1) Find \( \vec{A}(\vec{r}) = \frac{\mu e^{-j\beta r}}{4\pi r} \int e^{-j\beta r' r} \vec{J}(\vec{r}') d\vec{r}' \), where the integration is over the antenna surface (for linear antennas, the integration becomes line integral).

(2) Find \( \vec{E}(\vec{r}) = -j\omega \vec{A} \) (for \( \hat{\theta} \) and \( \hat{\phi} \) components)

(3) Find \( \vec{H}(\vec{r}) = \hat{r} \times \vec{E} / \eta \)

Example (ideal dipole)

\[
\vec{A} = \frac{\mu e^{-j\beta r}}{4\pi r} \int_{-\Delta z/2}^{\Delta z/2} e^{-j\beta r' z'} I z' dz' = \frac{\mu I \Delta z}{4\pi} \frac{e^{-j\beta r}}{r} \left( \hat{r} \cos \theta + \hat{\theta} \sin \theta \right)
\]

\[
\vec{E} = j\omega \mu \frac{I \Delta z}{4\pi} \frac{e^{-j\beta r}}{r} \sin \theta \hat{\theta}
\]

\[
\vec{H} = \hat{r} \times \vec{E} / \eta = \frac{I \Delta z}{4\pi} \frac{e^{-j\beta r}}{r} j\beta \sin \theta \hat{\phi}
\]

(note: \( \eta = \omega \mu / \beta \))
• Uniform Line Source

Length: L
Current density:
Uniform  \( I_0 \)
Location: -L/2 to L/2
Orientation: \( \hat{z} \)

Potential:

\[
\overline{A} = \frac{\mu I_0 e^{-j\beta r}}{4\pi r} \int_{-L/2}^{L/2} e^{j\beta z'} I_0 \hat{z} \, dz'
\]

\[
= \hat{z} \frac{\mu I_0 e^{-j\beta r}}{4\pi r} \int_{-L/2}^{L/2} e^{j\beta z' \cos \theta} \, dz'
\]

\[
= \hat{z} \frac{\mu I_0 e^{-j\beta r} e^{j\beta (L/2) \cos \theta} - e^{-j\beta (L/2) \cos \theta}}{4\pi r \beta \cos \theta}
\]

\[
= \hat{z} \frac{\mu I_0 L e^{-j\beta r}}{4\pi r \beta (L/2) \cos \theta} \sin \left( \beta (L/2) \cos \theta \right)
\]

since \( \hat{z} = \hat{r} \cos \theta + \hat{\theta} \sin \theta \)

\[
\overline{E} = \hat{\theta} \frac{j \omega \mu I_0 L e^{-j\beta r}}{4\pi r \sin \theta} \sin \left[ \frac{(\beta L/2) \cos \theta}{(\beta L/2) \cos \theta} \right]
\]

Radiation pattern:

\[
F_\theta(\theta, \phi) = \sin \theta \frac{\sin \left[ \frac{(\beta L/2) \cos \theta}{(\beta L/2) \cos \theta} \right]}{(\beta L/2) \cos \theta}
\]

Maximum radiation direction: \( \theta = \pi / 2 \).
Properties of "sinc()" function: \( \frac{\sin(x)}{x} \)

1. Maximum: \( x=0 \): level=1 (0 dB)
2. Half power point at \( x=1.3916 \) (Level=-3dB)
3. Zero-points at \( x = n\pi \), \( n=\text{integers} \)
4. First side lobe: \( x=4.493 \), Level=0.217 (-13.2dB)
5. Second side lobe: \( x=7.725 \), Level=0.128 (-17.8dB)
• **Radiation pattern parameters:**

![Diagram of radiation pattern](image)

- **Main lobe max. direction**
- **Side lobes**

**Side lobe level (SLL) is defined as:**

\[
SLL_{dB} = 20 \log \left| \frac{F(SLL)}{F(\text{max})} \right|
\]

**Half-power bandwidth:**

\[
HP = |\theta_{HP,\text{left}} - \theta_{HP,\text{right}}|
\]

**Main lobe direction vs. antenna orientation:**

**Endfire:** Main lobe parallel to antenna axis.

**Broadside:** Main lobe perpendicular to antenna axis.
Use MATLAB function "fzero" to find roots

For example, to find the half power point of the pattern \( \sin(2\sin\theta)/(2\sin\theta) \), we need to solve the following equation:

\[
\left| \frac{\sin(2\sin\theta_h)}{(2\sin\theta_h)} \right| = 0.707, \quad \theta_h = ?
\]

In MATLAB prompt ">", type

```matlab
fzero('abs(sin(2*sin(x))/(2*sin(x)))-0.707',[0.01,2])
```

Will get

x=0.7697

You can plot the function to estimate the initial solution region.