EE572
HW #7
Due Wednesday, February 10

1. a) If \( x_{k+1} = A x_k + B w_k \), find the eigenvalues and eigenvectors then use the similarity transformation, \( x_k = P z_k \), to determine which of the eigenvalues are controllable (you may use Matlab if you wish):

i) \( A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \)

ii) \( A = \begin{bmatrix} 1/2 & -2 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -4 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \)

b) For the above problem, find the rank of the controllability matrices and determine if the systems are completely controllable. Do your answers agree with the answers in part a)?

c) For problem 1a), state which of the eigenvalues are stable and which are unstable.

d) Find the region in the s-plane which corresponds to the following design specifications:
A settling time (2\%) of 0.2 sec and no overshoot/oscillations (recall \( \text{Re}[s]\max = -4/t_s \) defines the border of the settling-time region in the LHP of the S-Plane.)

e) Repeat part d) in the Z-plane if \( T_s = 10 \) msec. (map the above region using \( z = e^{sT} \))

2. Consider the following discrete-time state variable model:

\[
\begin{align*}
x_{k+1} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} w_k
\end{align*}
\]

a) Is the system completely controllable (use Matlab to find \( x_k = T z_k \) to decouple)?

b) Is the system stable?

c) If the system is completely controllable (which it is), design a feedback control law, \( w_k = -k x_k \) such that the closed-loop eigenvalues are \( \{.4,.4,.4\} \).

d) Complete the block diagram of your new closed-loop system (we drew this in class):

![System Diagram]

e) What is settling time of the new closed-loop system if \( T_s = 10 \) msec (hint: \( |z|\max = 0.4 \) after we set all the closed-loop eigenvalues. Solve \( |z|\max = e^{sT}\max \) for the settling-time, \( t_s \).