2. Given the system:

\[ W(s) + \frac{10}{s(s+8)} \rightarrow Y(s) \]

\[ T_s \]

\[ G_{zoh}(z) \]

\[ G_{na}(z) \]

a) Find an s-domain model for the open-loop system including the ZOH if \( T_s = 10 \) msec.

**Solution:** From class we learned that the approximate transfer function for \( G_{ZOH}(s) \) is \( \frac{2}{Ts/(2/Ts+s)} \). Substituting \( T_s = 10 \) msec. into this expression yields \( G_{ZOH}(s) = \frac{200}{s+200} \). Thus, the entire open-loop model is

\[ G(s) = \frac{1000}{s(s+8)(s+200)} \]

b) What type number is your S-domain model?

**Solution:** The system is a type 1 system (i.e., one open-loop pole at the origin)

c) Find \( e_{ss} \) due to a unit step, \( e_{ss} \) due to a unit ramp, and \( e_{ss} \) due to a unit parabola for your uncompensated model.

**Solution:** \( e_{ss|\text{unit step}} = \frac{1}{1+Kp} = 0 \), \( e_{ss|\text{unit ramp}} = \frac{1}{Kv} = \frac{(200)(8)/(2000)}{4/5} = \frac{4}{5} \), and \( e_{ss|\text{unit ramp}} = \frac{1}{Ka} = \infty \)

d) Given the following transient specifications: \( t_s < 0.4 \) sec and \( M_p < 2\% \). Illustrate the region of the s-plane and then the z-plane where we must place our dominant poles to satisfy these specs.

**Solution:** from the first spec, we find that \( t_s < 0.4 \Rightarrow \zeta \omega_n > 10 \). From the second spec we find that \( M_p = e^{\frac{\zeta}{\sqrt{1-\zeta^2}}} \times 100\% \). Or, solving this relationship for the damping coefficient we obtain, \( \zeta = \frac{\ln(m^2)}{\pi^2 + \ln(0.02)^2} = 0.7797 = \cos \theta \). Hence, \( \theta = \cos^{-1}(\zeta) = \cos^{-1}(0.7797) = 38.77^\circ \).

Since we must have \( M_p < 2\% \), our constraint becomes \( \theta \leq 38.77^\circ \). In the s-plane, we obtain the following region:

In the z-plane, this region maps to:

e) Sketch the root locus. Design \( G_{lead}(s) \) then find \( G_{lead}(z) \) (using the bilinear transformation) to meet the above specs.

**Solution:** Choose the desired dominant poles to be \( s_1 = -11+j6 \). The root locus of \( G_{ZOH}G(s) = \frac{2000}{s(s+8)(s+200)} \) looks...
like:  Obviously, the uncompensated root locus does not pass thru the desired poles. So, we need to insert a lead compensator of the form $G_{\text{lead}}(s) = K_c \frac{s + z_c}{s + p_c}$. Our first step in designing this compensator is to find the angle of deficiency (i.e., the amount of lead angle needed to bend the root locus thru $s_k$). Find the angle of deficiency $\angle G_{\text{lead}}(s_k) = 180^\circ \times \text{odd} - \angle G_{\text{coh}}G(s_k)\mid_{s_k=-11+j6} = 180^\circ \times \text{odd} - 90.23^\circ = 89.77^\circ$. Therefore, our lead compensator must supply 89.77 degrees.

Let's pick $z_c$ to be as far to the left as possible. Let $z_c$ be 10.8 (by geometry, $z_c$ must be less than or equal to 11). Then $\angle G_{\text{lead}}(s_1) = 89.77^\circ = \angle(s_1 + 10.8) - \angle(s_1 + p_c) = 91.9^\circ - \angle(s_1 + p_c)$ or $\angle(s_1 + p_c) = 213^\circ$. Solving for $p_c$: $\text{IM}(s_1) / \text{RE}(s_1 + p_c) = \tan(213^\circ)$ or $p_c = \text{IM}(s_1) / \tan(213^\circ) - \text{RE}(s_1) = 171.85$. The last step is to find $K_c$ from the magnitude condition: $K_c = 1 / [G_{\text{coh}}G(s_1)(s_1 + z_c) / (s_1 + p_c)]^\circ = 213.08$. Thus, the lead compensator is $G_{\text{lead}}(s) = 213.08(s+10.8)/(s+171.85)$. Finally, the digital filter design can be found using the bilinear transformation: $G_{\text{lead}}(z) = G_{\text{lead}}(s)\mid_{s=2(z-1) / (z+1)} = 153.4(z-0.8975)/(z-0.0757)$

e) Use Matlab to determine where all the closed-loop compensated poles and zeros are (Hint: Find the closed-loop compensated transfer function, $Y(s)/W(s)=G_{\text{lead}}G_{\text{coh}}G(s)/(1+G_{\text{lead}}G_{\text{coh}}G(s))$ then use the Matlab roots() function to find the poles and zeros) 

**Solution:** the following commands in Matlab can be used to find the closed-loop transfer function, $G_{\text{lead}}G_{\text{coh}}G(s)/(1+G_{\text{lead}}G_{\text{coh}}G(s))$:

```matlab
» num=2000*kc*[1 z]
num =
1.0e+006 *
0.4262    4.6024

» den=poly([0,-8,-200,-p])
den =
```

```
1.0e+005 *
  0.0000  0.0038  0.3734  2.7496  0
» [num_closed_loop,den_closed_loop]=cloop(num,den)
num_closed_loop =
1.0e+006 *
  0  0  0  0.4262  4.6024
den_closed_loop =
1.0e+006 *
  0.0000  0.0004  0.0373  0.7011  4.6024
» roots(den_closed_loop)
an =
1.0e+002 *
-2.3088
-1.2697
-0.1100 + 0.0600i
-0.1100 - 0.0600i

f) Sketch the compensated root locus in the S-plane.
Solution: The compensated root locus looks like:

![Root Locus Diagram]

g) Does your compensated system have closed-loop dominant poles which meet the specifications?
Solution: as can be seen from the Matlab output, the closed-loop poles are \{-11+j6, -11-j6, -126.97, -230.88\}. So, yes the dominant poles are at exactly $s_1=-11+j6$. 