0. Check your scores under the grades option and make sure all records are correct!

1.a) Sketch the root locus of the following s-plane open-loop pole zero. **Solution:**

i) (two poles at the origin)

ii) (two poles)

iii) (two zeros)

1b) Show the region of the s-plane where we must place our dominant poles to satisfy the following specifications:

i) \( t_s \leq 0.5 \) sec and \( M_p \leq 50\% \)

ii) \( t_s \leq 1 \) sec and \( z \leq 0.707 \)

i) **Solution:** from the first spec, we find that \( t_s \leq 0.5 \Rightarrow \zeta \omega_n \geq 8 \). From the second spec we find that \( M_p = e^{\frac{-\zeta \pi}{\sqrt{1-\zeta^2}}} \times 100\% \). Or, solving this relationship for the damping coefficient we obtain, \( \zeta = \frac{\ln(m) \sqrt{2}}{\sqrt{\pi^2 + \ln(m)^2}} \) where \( m = M_p/100\% \). Thus, \( \zeta = \frac{\ln(0.5) \sqrt{2}}{\sqrt{\pi^2 + \ln(0.5)^2}} = 0.2155 = \cos \theta \). Hence, \( \theta = \cos^{-1}(\zeta) = \cos^{-1}(0.2155) = 77.56° \). Since we must have \( M_p < 50\% \), our constraint becomes \( \theta \leq 77.56° \). Putting both of these constraints together, we find the region in the s-plane shown:

ii) **Solution:** from the first spec, we find that \( t_s \leq 1.0 \Rightarrow \zeta \omega_n \geq 4 \). From the second spec we find that \( \zeta = \cos \theta \leq 0.707 \). Hence, \( \theta = \cos^{-1}(\zeta) \geq \cos^{-1}(0.707) = 45° \). Putting both of these constraints together, we find the region in the s-plane shown:

1c) **Solution:** plugging in the region \( -\sigma = \zeta \omega_n \geq 8 \) into our mapping \( z = e^{st} \) produces the following circle within the unit circle with a
radius of \( z_{\text{max}} = e^{-8x0.1} = 0.449 \). Mapping the region corresponding to \( s = r \angle \theta \) where \( \theta \geq 180^\circ - 87.34^\circ \) produces the logarithmic spiral shown:

By intersecting these two areas, we find the complete solution shown:

ii) For part ii), the settling time condition \( t_s \leq 1.0 \Rightarrow \zeta \omega_n \geq 4 \) produces the circle within the unit circle shown with a radius of \( z_{\text{max}} = e^{-4x0.1} = 0.67 \):

The region defined \( \theta = \cos^{-1}(\zeta) \geq \cos^{-1}(0.707) = 45^\circ \) maps into the region shown:

Intersecting these two regions produces the following region in the \( z \)-plane:

2a)  
   i) \textbf{Solution:} \( e_{\text{sol}}=1/(1+K_p) \). Thus, \( e_{\text{sol}|10u(t)}=10/(1+K_p) = 10/(1+12/5)=50/17 \)  
   ii) \textbf{Solution:} \( e_{\text{sol}|10u(t)}=1/K_c \). Thus, \( e_{\text{sol}|10u(t)}=10/K_c = \infty \)  
   iii) \textbf{Solution:} \( e_{\text{sol}|10u(t)}=1/K_s \). Thus, \( e_{\text{sol}|10u(t)}=10/K_s = \infty \) (where \( p(t) = 0.5t^2u(t) = \text{a unit parabola} \)

2b)  
   i) \textbf{Solution:} \( e_{\text{sol}|10u(t)}=1/(1+K_p) \). Thus, \( e_{\text{sol}|10u(t)}=10/(1+K_p) = 0 \)  
   ii) \textbf{Solution:} \( e_{\text{sol}|10u(t)}=1/K_c \). Thus, \( e_{\text{sol}|10u(t)}=10/K_c = 50/12 = 25/6 \)  
   iii) \textbf{Solution:} \( e_{\text{sol}|10u(t)}=1/K_s \). Thus, \( e_{\text{sol}|10u(t)}=10/K_s = \infty \) (where \( p(t) = 0.5t^2u(t) = \text{a unit parabola} \)

2c) Matlab output:  
   ```matlab  
   » ahat=[1 0.0302;0 0.9487];  
   » bhat=[0.0104;0.6627];  
   ```
According to the separation principle, the eigenvalues should $\text{eig}(\hat{A} - \hat{B}K_e) \cup \text{eig}(\hat{A} - K_p\hat{C})$ which are $\{0.78, 0.76\} \cup \{0, 0\}$. As can been seen from the Matlab output, this is the case.