1.a) Sketch the root locus of the following s-plane open-loop pole zero configurations (Use Matlab's rlocus() command to check your answers):

i) -4 (two poles at the origin)

ii) -5-2 (two poles)

iii) -5 (two zeros)

b) In class we learned about the concept of dominant closed-loop poles and that we want to design a filter (compensator) to produce closed-loop poles that obey the equation $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$ or $s_{1,2} = -\zeta\omega_n \pm j\omega_n (1-\zeta^2)^{1/2}$. Show the region of the s-plane where we must place our dominant poles to satisfy the following specifications (hint: see cheat sheet II for the transient specification equations relating settling time and overshoot to $\zeta$ and $\omega_n$):

i) $t_s \leq 0.5$ sec and $M_p \leq 50\%$

ii) $t_s \leq 1$ sec and $\zeta \leq 0.707$

Fun Fact: Did you know that constant $\zeta$ (damping) lines in the s-plane map into logarithmic spirals in the z-plane? A point on a constant line can be described by the equation $s_1 = re^{\theta}s$ where $\cos\theta = \zeta = constant$ and the radius $r$ varies from 0 to $\infty$ (the -r indicates that the point is in the LHP). If we use the mapping $z = e^{ts}$, then these damping lines map to $z_1 = e^{s_1Ts} = e^{r\cos\theta Ts + jrsin\theta Ts}$ which in polar coordinates has a radius of $e^{-r\cos\theta Ts}$ and an angle of $r\sin\theta Ts$ (recall both $\cos\theta Ts$ and $\sin\theta Ts$ are constant). Thus, as we vary $r$ from 0 to $\infty$ in the s-plane, the point $z_1$ will start at $(1,0)$ in the z-plane and its radius will decrease exponentially while its angle will increase linearly. Such a plot is called a logarithmic spiral and looks like:

![logarithmic spiral](image)

2a) Given a unity feedback system with the open-loop transfer function $G(s) = \frac{6(s+2)}{s^m(s+5)}$, where the system type number $m=0$,

find (hint: remember the transfer function, $E(s)/W(s)$, is LINEAR so if we multiply $w(t)$ by a scalar, $e(t)$ is multiplied, too!):

i) $e_u$ due to 10u(t)  

ii) $e_u$ due to 10r(t)  

iii) $e_u$ due to 5t

b) Repeat part b) for a type 1 system.

c) Go back to your prelab #3, and check the eigenvalues of your combined full-order observer/controller design. That is, find the eigenvalues of the 2n$^h$ order system:

$$
\begin{bmatrix}
    x_{k+1} \\
    \dot{x}_{k+1}
\end{bmatrix} =
\begin{bmatrix}
    A & -BK \\
    K_\alpha \hat{C} & \hat{A} - K_\alpha \hat{C} - BK
\end{bmatrix}
\begin{bmatrix}
    x_k \\
    \dot{x}_k
\end{bmatrix}
$$

Does your answer agree with the separation principle?