Objective: The objective of Lab 2 is to: 1) complete our measurements of the MOTOMATIC internal parameters; 2) To obtain a state variable model of the MOTOMATIC.

Prelab (Counts as HW#9): (due Monday, October 3rd):

1. a) Using the transfer function \( \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} \) as found in Lab #1, find the SFG for the MOTOMATIC then find the state variable model in phase variable form.

b) The actual variables (voltages) we can measure are \( V_{\text{out}} \) (the voltage from the potentiometer wiper arm) and \( V_{g} \) (the voltage from the generator/tachometer). Recall that \( V_{g} = K_{g} \dot{\theta}_{m} = K_{g} N_{a} \dot{\theta}_{\text{out}} = K_{g} (9) \dot{\theta}_{\text{out}} \). Use a similarity transformation relating the state variables, \([V_{\text{out}} \ V_{g}]\) to the state variables you found in part a). Then, find a new state variable model in terms of the state variables, \([V_{\text{out}} \ V_{g}]\)

Hint: recall from class that we can use any similarity transform, \( x=Tz \), to transform the given state coordinates \( (x) \) into a new state coordinate \( (z) \). The result we derived was

\[
\dot{z} = T^{-1}ATz + T^{-1}Bw \\
y = CTz
\]

For this problem, \( x \) are the variables from your simulation diagram and \( z = \begin{bmatrix} V_{\text{out}} \\ V_{g} \end{bmatrix} \). When you find the similarity transformation matrix \( T \) where \( x=Tz \), \( T \) should be a diagonal matrix with the first element relating \( x_{1} \) to \( V_{\text{out}} \) and the second diagonal element relating \( x_{2} \) to \( V_{g} \).

2. The schematic for the closed-loop unity feedback MOTOMATIC DC servo for which you found the step response is shown in Figure 1. Note how the output voltage, \( V_{\text{out}} \), is fed-back to form an error (difference) signal with the system input, \( V_{\text{in}} \).

![Figure 1. Schematic of MOTOMATIC DC servo motor system.](image)

a) Find the functional block diagram from the above schematic with outputs \( V_{g} \) and \( V_{\text{out}} \).

b) Find the closed-loop transfer function, \( \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} \), in terms of the parameters \( J, F, R, \) etc.

c) Now let \( L_{a} = 0 \), and repeat part b. You should get a 2nd order system.

d) Finally, find the open-loop transfer functions, \( \frac{V_{g}(s)}{V_{\text{in}}(s)} \) and \( \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} \) (remember how we do open-loop transfer functions)
Ra and La If we hold the motor shaft so that it cannot move, then the back emf voltage must be zero (i.e., \( V_b = 0 \)). Find the transfer function, \( \frac{I_a(s)}{V_a(s)} \), when \( V_b = 0 \). If we apply a sinusoidal voltage with amplitude \( A \) and frequency \( \omega \) rad/sec at the armature terminals (i.e., \( V_a = A \sin \omega t \)), then \( I_a \) will also be a sinusoid in steady-state. Use your transfer function, \( \frac{I_a(j\omega)}{V_a(j\omega)} \) to find two equations involving \( R_a \) and \( L_a \) if we assume sinusoidal steady-state (Hint: think magnitude and phase).

In Lab (Counts as HW#10) (due Wednesday, October 5th)

In the lab, you will measure \( R_a, L_a, J, F, \) and \( K_a \).

1. a) Recall from EE415, that the value for the torque constant of a DC armature controlled motor is the same as the value for the back emf constant. Use your answer from Lab 1 for the back emf constant (\( K_b \)) to find the torque constant, \( K_T \) (make sure your units are consistent).

b) Apply a sinusoidal voltage at frequency \( \omega \) to the motor while holding the motor shaft to insure that \( V_b = 0 \). Take a plot of \( V_a \) and \( I_a \) versus time and find the magnitude and phase of \( \frac{I_a(j\omega)}{V_a(j\omega)} \). Use your answer to part 2e) of the Prelab to obtain values for \( R_a \) and \( L_a \).

c) One of the reasons that our MOTOMATIC is not truly a linear second-order system is that the friction present in the motor contains Coulomb friction as well as viscous friction. Coulomb friction (also known as static friction or "stiction") is approximately a constant force always opposing the motion of the motor. If we call this Coulomb friction constant \( C \), then we can write a new Newton's second law of motion for the Torques as seen at the motor:

\[
T_m = K_T I_a = J \omega_m + F \omega_m + C \text{sgn}(\omega_m) \tag{1}
\]

where \( \text{sgn}() \) is the signum function (i.e., \( \text{sgn}(x) = +1 \) if \( x > 0 \) or \( -1 \) if \( x < 0 \)).

One technique for measuring parameters is called a steady-state plot. This technique is performed by inputting a constant into the system, waiting for the system to reach steady-state, then measure the system's output. By repeating this procedure for a variety of constant inputs, a plot of steady-state output values versus constant input values is obtained. For our particular system, we will input a constant \( V_{in} \) which will produce a constant motor speed (i.e., \( \omega_m = \) constant). If we measure the generator output voltage, \( V_g = K_g \omega_m \), then we can write an equation for \( I_a \) in terms of \( V_g \) from equation (1). In fact, this equation will be the equation of a straight line \( (y = mx + b) \). Take a steady-state plot of \( I_a \) vs. \( V_g \). Your plot should be fairly linear. Use the slope and the y-intercept of this plot to find the viscous friction coefficient, \( F \), and the Coulomb constant, \( C \).

2. a) If we ignore the nonlinear Coulomb friction (assume \( C = 0 \)), then equation (1) reduces to:

\[
T_m = K_T I_a = J \omega_m + F \omega_m = J V_g / K_g + F V_g / K_g \tag{2}
\]

Run the motor at a constant speed, then set \( I_a = 0 \) by pulling out the lead of the armature circuit of the MOTOMATIC while the motor is turning. This is accomplished by pulling the red cap at the terminal labelled A (Please pull the cap not the red wire!). Measure the values of \( V_g \) and derivative of \( V_g \) at the point on the plot where you set \( I_a = 0 \). Then, use equation (2) to find the motor inertia, \( J \).

b) Make a plot of \( V_a \) vs. \( V_{in} \). Find \( K_a \), the gain of the amplifier, from this plot.
c) Substitute the values of \( K_b, K_T, K_p, R_a, L_a, F \) and \( J \) into your answers for Prelab problems 2b) and 2c). Compare your answers from Lab 1 for \( V_{out}(s)/V_{in}(s) \) as measured from the step response in experiment 1. Which modelling technique more accurately models how \( V_{out} \) responds to \( V_{in} \), the step response method or internal parameter measurement? What are the advantages of each modelling technique?