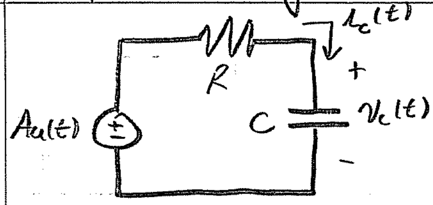
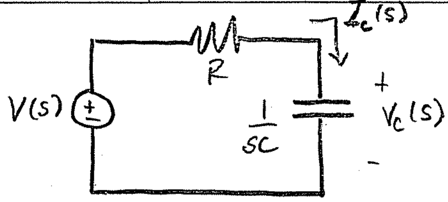
Stephen Maloney | EE462 Pre-lab 1

1.





<u>KVL - Time Domain</u>

$$-Au(t) + i_c R + v_c = 0 \qquad i_c = C\frac{dv_c}{dt}$$

$$RC\frac{dv_c}{dt} + v_c = Au(t)$$

$$\frac{dv_c}{dt} + \frac{v_c}{RC} = \frac{A}{RC}u(t)$$

$$e^{t/RC}\left[\frac{dv_c}{dt} + \frac{v_c}{RC}\right] = \frac{A}{RC}e^{t/RC}u(t)$$

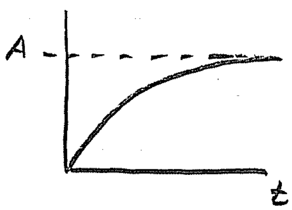
$$\frac{d}{dt}\left[e^{t/RC}v_c\right] = \frac{A}{RC}e^{t/RC}u(t)$$

$$e^{t/RC}v_c = \frac{A}{RC}\int_0^t e^{t/RC}\,dt$$

$$e^{t/RC}v_c = \frac{A}{\cancel{RC}}\cancel{(RC)}\left[e^{t/RC}\right]_0^t$$

$$e^{t/RC}v_c = A(e^{t/RC} - 1)$$

$$v_c(t) = A(1 - e^{-t/RC})\,u(t)$$



<u>Voltage Division - s Domain</u>

$$V_c(s) = \frac{V(s)\frac{1}{sC}}{\frac{1}{sC} + R} = \frac{V(s)}{1 + sRC}$$

$$V(s) = \mathcal{L}(Au(t)) = \int_{-\infty}^{\infty} Au(t)\,e^{-st}\,dt$$

$$V(s) = A\int_0^{\infty} e^{-st}\,dt = -\frac{A}{s}\left[e^{-st}\right]_0^{\infty}$$

$$V(s) = \frac{A}{s}$$

$$V_c(s) = \frac{A}{s(1+sRC)} = \frac{A/RC}{s(\frac{1}{RC}+s)}$$

$$A/RC\left[\frac{1}{s(\frac{1}{RC}+s)}\right] = \frac{A}{RC}\left[\frac{D}{s} + \frac{E}{\frac{1}{RC}+s}\right]$$

$$D = \left.\frac{1}{\frac{1}{RC}+s}\right|_{s=0} \qquad E = \left.\frac{1}{s}\right|_{s=-\frac{1}{RC}}$$

$$D = RC \qquad E = -RC$$

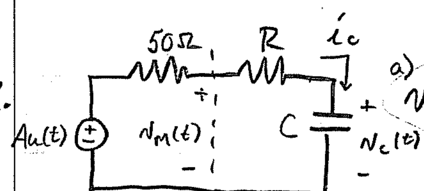
$$V_c(s) = \frac{A}{RC}\left(\frac{RC}{s} - \frac{RC}{\frac{1}{RC}+s}\right)$$

$$V_c(s) = \frac{A}{s} - \frac{A}{\frac{1}{RC}+s}$$

$$v_c(t) = Au(t) - Ae^{-t/RC}u(t)$$

$$v_c(t) = A(1 - e^{-t/RC})\,u(t)$$

2.



a) $v_c(t) = A(1 - e^{-t/(R+50)C})\,u(t) \rightarrow \tau = (R+50)C$

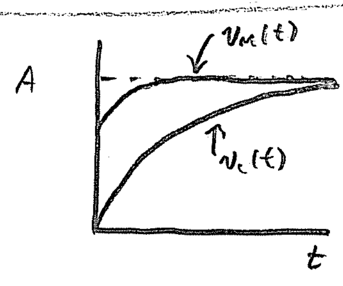
b) $v_M(t) = Ri_c(t) + v_c(t) = RC\frac{dv_c}{dt} + v_c(t)$

$$\frac{dv_c}{dt} = (-Ae^{-t/\tau})\left(\frac{-1}{\tau}\right)$$

$$\frac{dv_c}{dt} = \frac{A}{\tau}e^{-t/\tau}$$

$$v_M(t) = \frac{ARC}{\tau}e^{-t/\tau} + A(1 - e^{-t/\tau})$$

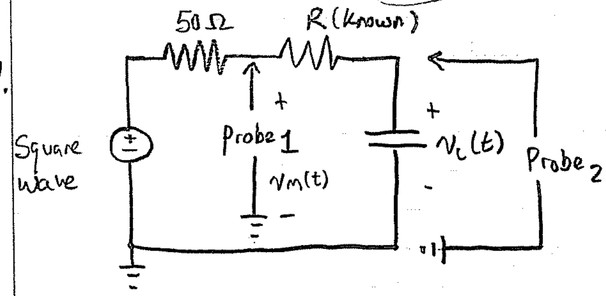
$$v_M(t) = A\left[1 - e^{-t/\tau} + \frac{RC}{\tau}e^{-t/\tau}\right]$$

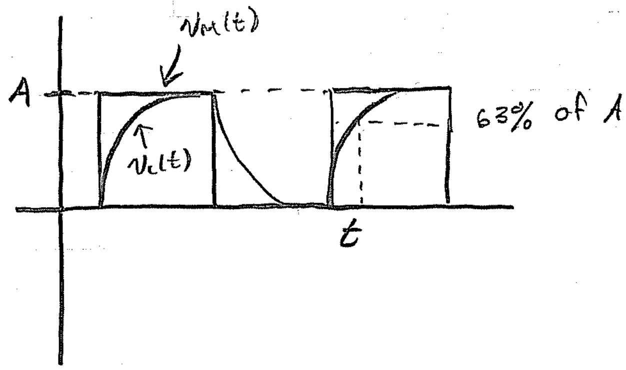


3. $v_c(t) = A\left(1 - e^{-t/RC}\right)u(t)$ $\qquad R = 200\,\Omega$, $A = 5\,V$, $v_c(10ms) = 3.2\,V$

$$3.2 = 5\left(1 - e^{-(10ms)/200C}\right)$$
$$.64 = 1 - e^{-5\times10^{-5}/C}$$
$$.36 = e^{-5\times10^{-5}/C}$$
$$C\ln(.36) = -5\times10^{-5}$$
$$C = 4.894\times10^{-5} \approx 49\mu F$$

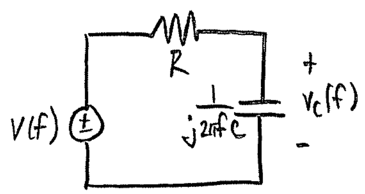
4.





- Set square wave input with offset to be positive only
- Set frequency of square wave to allow for capacitor to charge and discharge sufficiently.
- Find using cursors when cap voltage is 63% of square wave max.
- The time from start of the pulse to this time is RC → solve for C.

5.



$$V_c(f) = \frac{V(f)\frac{1}{j2\pi fC}}{R + \frac{1}{j2\pi fC}} = \frac{V(f)}{1 + j2\pi fRC}$$

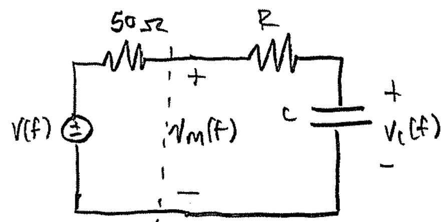
$$H(f) = \frac{1}{1 + j2\pi fRC} = \frac{V_c(f)}{V(f)} = \frac{\text{out}}{\text{in}}$$

$$|H(f)| = \frac{1}{|1 + j2\pi fRC|} = \frac{1}{\sqrt{1^2 + (2\pi fRC)^2}} = \frac{1}{\left(1 + (2\pi fRC)^2\right)^{1/2}}$$

$$\angle H(f) = \frac{\angle 1}{\angle 1 + j2\pi fRC} = \angle 1 - \angle 1 + j2\pi fRC$$

$$\angle H(f) = -\tan^{-1}\left(\frac{2\pi fRC}{1}\right) = -\tan^{-1}(2\pi fRC)$$

6.



If $V_m(f)$ is reference, we have the circuit of #5, and thus the same TF.

7. $$|V_c| = \frac{|V|}{\left(1 + (2\pi f RC)^2\right)^{1/2}}$$

$$|V_c|^2 = \frac{|V|^2}{1 + (2\pi f RC)^2}$$

$$\cancel{1+} (2\pi f RC)^2 = \frac{|V|^2}{|V_c|^2} - 1$$

$$\cancel{2\pi f RC} = \frac{\sqrt{\frac{|V|^2}{|V_c|^2} - 1}}{2\pi f R}$$

$$C = \frac{\sqrt{\frac{|V|^2}{|V_c|^2} - 1}}{2\pi f R} = 5.412 \times 10^{-7}\ F \approx .54\mu F$$

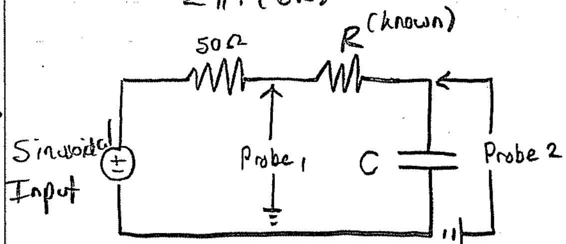
8. $$\angle H(f) = -\tan^{-1}(2\pi f RC)$$

$$-\frac{\tan(\angle H(f))}{2\pi f R} = C$$

$$C = \frac{-\tan(\angle V_c - \angle V)}{2\pi f R}$$

$$C = \frac{-\tan(-45°)}{2\pi f (6k)} = 6.63 nF$$

9.



Magnitude

Measure input/output as shown above; the peak/peak measurements can be used in the formula:

$$C = \frac{\sqrt{\left(\frac{Vs(pp)}{Vc(pp)}\right)^2 - 1}}{j 2\pi f R}$$

Phase

Run a frequency sweep and find where the phase difference is 45°.

$$\angle H(f) = -\tan^{-1}(2\pi f RC) \qquad \angle V_S \text{ reference} = 0°$$

$$\angle V_c - \angle V_S = -\tan^{-1}(2\pi f RC)$$

$$-\tan(\angle V_c) = 2\pi f RC$$

$$C = \frac{-\tan(\angle V_c)}{2\pi f R} = -\tan(-45°) = \frac{1}{2\pi f R} = C$$

At 45°, $C = \frac{1}{2\pi f R}$