Laboratory #1: Measuring Capacitance

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Objectives

- Introduce lab instrumentation with linear circuit elements
- Introduce lab report format
- Develop and analyze measurement procedures based on two theoretical models
- Introduce automated lab measurements and data analysis

Procedure

The goal of this experiment is to calculate an unknown capacitance in a simple RC circuit using two different theoretical models: the circuit's step and frequency responses. **Figure 1** details the circuit used in this experiment.

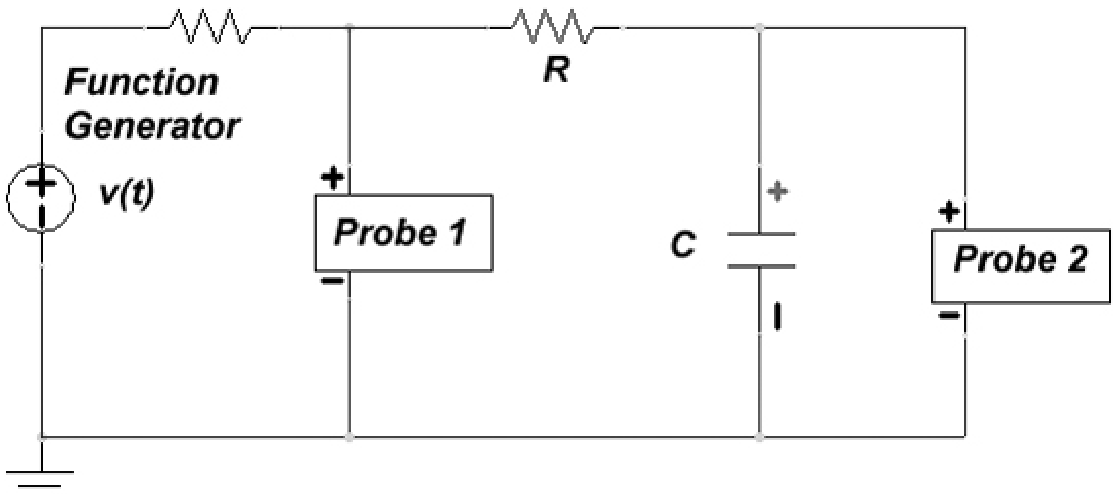

Figure 1: RC Circuit Used Through Laboratory 1

1: Step Response Model

When a voltage step is applied to a series RC circuit the voltage across the capacitor will change according to the equation

$$v_c(t) = A\left(1 - e^{-\frac{t}{RC}}\right)$$

where A is the amplitude of the step, R is the resistance of the resistor, C is the capacitance of the capacitor, and *t* is the time in seconds after application of the step. One can take advantage of this relationship to determine an unknown capacitance by applying a step of known magnitude to a circuit with known resistance and measuring the voltage across the capacitor at a particular time, thus making C the sole dependent variable of the step response equation.

In order to accomplish this, the above circuit was constructed, using the oscilloscope channel 1 to measure the input voltage across an arbitrary resistor and the given unknown capacitor, and using channel 2 to measure the voltage across the capacitor. The probes share a common ground and must have all ground clips connected together to prevent shorting elements out of the circuit. Care must be taken when selecting the resistor value – too large a value and no output voltage will be read across the capacitor, too low a value and the internal resistance of the function generator will cause voltage division effects. It is also important to measure the value of the resistor using a multimeter to account for the inherent variance in resistor values.

Once the circuit was constructed and the oscilloscope and function generator were connected as in Figure 1, the function generator was configured to output a square wave 1 kilohertz. The square wave is used to simulate a step response, and must have an adequate period to give the capacitor time to reach an approximately steady state. In order to do this, the display of the oscilloscope must be viewed and the frequency adjusted on the function generator until the channel 2 wave form rising exponential flattens out. Once this is accomplished, adjusting the wave forms until the square wave oscillates between 0 and the maximum voltage, and then adjusting the output waveform to fit inside this square wave, matching the maximum and minimum points. See Figure 2 below for an example snapshot.

Once the waveforms are displayed in this manner, the cursors can be used to find time in seconds (along the horizontal) since the start of the square wave to the point that the capacitor has charged to 63%. This is equal to one time constant, the product of the R and C values. By dividing this value by the known resistance value, the unknown capacitor value can be found.

This part of the experiment was repeated for four different arbitrary resistor values.

2: LabView Preparation

LabView allows the automation and control of both the oscilloscope and function generator. In order to set up for automated measurements required for the frequency response model, a set of four resistors, chosen this time similarly as above were placed in the circuit, one at a time. LabView requires the setting of frequency, amplitude, and function type to operate and outputs the magnitude of the voltage readings for both channels, and the phase difference between the two. To use the frequency response model the function must be a sine wave. After setting this, the object for each resistor is to find the frequency at which the phase difference is approximately 45°. This is the point that the transfer function has reached what is known as the cutoff frequency – the point where the product of the radial frequency, resistor, and capacitor values are 1.

$$H(\omega) = \frac{1}{1 + j\omega RC}$$

To find this point, make a cursory measurement. If the phase difference is greater than 45°, adjust the frequency down for the next attempt, and if the phase difference is below 45°, adjust the frequency up.

3: LabView Automation

Now that the cutoff frequency for each resistor value is approximated, LabView can be used to measure many values around this area and provide the data necessary to do the data analysis required. For each resistor, an ASCII file was created with a list of frequencies separated by carriage returns. These frequencies varied around the approximate cutoff frequency found above by a decade, and were concentrated around this approximation.

For the trials, an amplitude of four volts was used to prevent voltage saturation from making the phase determination difficult. LabView then produced all the output data at these frequencies, which were then saved. These separate files are conveniently in the format as seen in the pre-lab, and the Matlab script written was used to do the curve fitting and produce a best fit cutoff frequency. This frequency can then be used to find the capacitor value by

$$\frac{1}{RC} = \omega$$

$$C = \frac{1}{R\omega}$$

Finally, these capacitor values can be used to find confidence limits. By doing a magnitude, phase, and a combination of the two fit, the confidence limits can be narrowed down significantly.

Presentation of Results

1. Capacitance from Step Response

In the first part of the experiment the step response of the series RC was observed in order to calculate the capacitance of the capacitor. An example of the oscilloscope's output for the setup is shown in Figure 2.

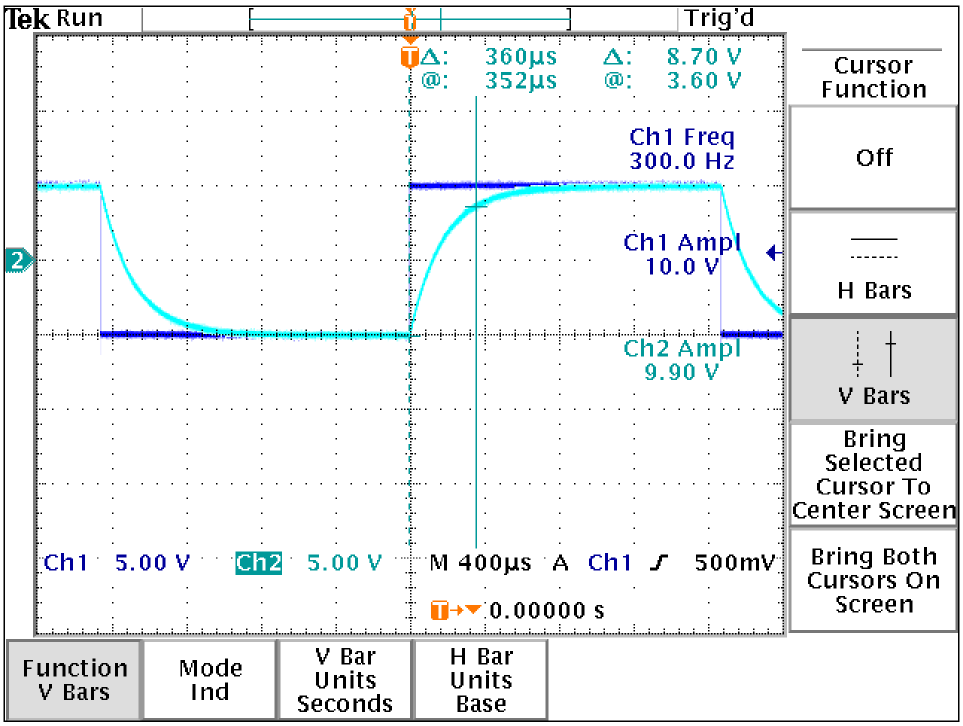

Figure 2: Oscilloscope Reading for a Series RC Circuit. Ch 1 is V_{in} (a square wave) and Ch 2 is V_c (a decaying exponential).

In order to find the capacitance of the capacitor we chose four different resistors to place in the circuit and, for a certain frequency of a 10-V_p square wave, measured the length of time required for the capacitor to charge to roughly 6.3V. (Any time-voltage pair will do; it is adequate to choose a target that would be roughly one time constant after the start of the charging cycle.) These measurements, along with the computed capacitances for each trial, are shown in **Table 1**.

Resistance (kΩ)	Frequency (Hz)	Capacitor Voltage (V)	Time to Reach Voltage (µs)	Capacitance (nF)
9.8	300	6.3	328	33.47
2.17	800	6.2	74	34.1
21.7	100	6.1	780	35.94
5.07	100	6.3	180	35.5

Table 1: Voltage Measurements for a Series RC Step Response Circuit and Corresponding Capacitances

For each of these measurements in Table 1 the capacitance was computed from the equation for the step response of an RC circuit:

$$v_c(t) = A\left(1 - e^{-\frac{t}{RC}}\right)$$

For example, the first measurements from Table 1 yield the indicated capacitance through the following substitution:

$$v_c(t) = 10\left(1 - e^{-\frac{328\times10^{-6}}{(9800)C}}\right)$$

$$C = 33.47nF$$

2. Capacitance from Magnitude and Phase of Frequency Response

For the second part of the experiment a LabView program was used to perform a frequency sweep of a series RC circuit. We began by using the program test_use_keyboard.exe in order to find an AC frequency for each RC circuit that would yield a roughly 45° difference in phase between the source and capacitor voltage waveforms. These results are displayed in Table 2.

Resistance (kΩ)	Frequency (Hz)	Phase (degrees)
5.07	937	45.534
1.96	2500	45.596
11	420	44.896
2.18	2200	45.552

Table 2: Series RC Circuit Phase Measurements for Several Resistances and Input Frequencies

The values in Table 2 served as references around which to choose frequencies that would give meaningful frequency response information, because since the phase difference varies as the inverse tangent of the frequency we wanted the majority of our input frequencies to be around or below the frequency yielding 45 degrees so that our results would have a good spread of phases for the curve fit. For each resistance a list of frequencies was created in a test file and fed into the LabView program "test_use_files_freq.exe", which read the magnitude of the input and output sinusoids and the phase difference between the signals for each frequency. These results are shown in Table 3, Table 4, Table 5, and Table 6. These data were then fed into a Matlab script in Appendix A that iterates over several values of theoretical cutoff frequency, f_c = 1/2πRC, and selects the one that provides the smallest mean-square error for a computed set of points versus the experimental data for both the measured phase and ratio of V_{in} to V_{out}. The results of running this program on the four sets of data are shown in Table 7.

Frequency (Hz)	|Vin| (V)	|Vc| (V)	Phase (degrees)
250	8.042	8.0197	5.423
800	8.0198	7.6547	18.568
1600	8.0086	6.697	32.801
2000	7.9295	6.1716	39.61
2100	7.9328	6.0559	41.123
2200	7.9437	5.9337	42.742
2300	7.9345	5.8178	43.598
2400	7.9295	5.7025	44.539
2500	7.9209	5.5784	46.586
2600	7.9117	5.4875	46.853
2700	7.9062	5.3231	48.402
2800	7.9228	5.2481	49.439
2900	7.9209	5.1319	50.085
5000	7.8909	3.5409	62.498
10000	7.8692	1.9202	73.949
15000	7.8686	1.3466	77.405
20000	7.8766	1.0236	83.8
25000	7.8467	0.85	81.084

Table 3: Voltage and Phase Data, R=1.96 kOhms

Frequency (Hz)	|Vin| (V)	|Vc| (V)	Phase (degrees)
220	8.0386	8.0042	6.0469
800	8.0159	7.5661	20.247
1600	7.9987	6.4687	36.799
2000	7.9686	5.8972	43.062
2100	7.9306	5.7784	43.353
2200	7.9412	5.6347	45.137
2300	7.9408	5.5056	46.445
2400	7.9427	5.3778	47.478
2500	7.9264	5.2472	49.31
5000	7.9073	3.245	65.845
10000	7.8787	1.7283	76.771
15000	7.8895	1.2112	79.16
20000	7.885	0.9375	80.525
22000	7.8884	0.86	79.513

Table 4: Voltage and Phase Data, R=2.18 kOhms

Frequency (Hz)	|Vin| (V)	|Vc| (V)	Phase (degrees)
93	8.0573	8.0241	5.2899
200	8.05	7.965	12.172
400	8.0402	7.4442	23.535
600	8.0316	6.7784	32.885
800	8.0198	6.105	40.793
900	8.0178	5.7953	44.309
920	8.0158	5.7266	44.6
930	8.0067	5.7078	45.238
937	8.0127	5.6962	45.415
950	8.008	5.6356	45.909
960	8.0106	5.6144	46.214
970	8.0028	5.595	46.724
980	8.0186	5.5662	46.606
1000	8.018	5.5037	46.854
2000	7.9672	3.4472	64.36
4000	7.9858	1.8911	75.688
6000	7.9894	1.3166	79.038
8000	7.9845	1.012	79.578
9000	7.9811	0.9044	80.273
9370	7.9811	0.8739	79.361

Table 5: Voltage and Phase Data, R=5.07 kOhms

Frequency (Hz)	|Vin| (V)	|Vc| (V)	Phase (degrees)
42	8.0131	7.9462	5.7232
55	8.017	8.0306	7.0608
80	8.0373	8.0161	10.038
100	8.0345	7.9181	13.205
120	8.0375	7.7902	15.928
200	8.0322	7.3317	25.874
300	8.0347	6.6244	34.929
350	8.0295	6.2466	39.786
410	8.0253	5.8122	43.62
420	8.0275	5.7762	44.445
430	8.0214	5.6856	44.784
450	8.0277	5.5412	46.318
470	8.0258	5.4484	47.845
500	8.0291	5.2356	49.038
1000	8.0277	3.2084	65.731
2000	7.9966	1.7298	76.855
3000	8.0128	1.1964	81.514
4000	8.0392	0.9411	80.678
4200	8.0031	0.9052	80.356

Table 6: Voltage and Phase Data, R=11 kOhms

Resistance (kΩ)	Estimated f_c (Mag, Hz)	Estimated f_c (Phase, Hz)	C (Mag, nF)	C (Phase, nF)
2.18	2225	2206	32.812	33.093
11	435	432	33.261	33.492
1.96	2486	2442	32.664	33.252
5.07	949	937	33.079	33.502

Table 7: Estimated Cutoff Frequencies and Associated Capacitances from Matlab Script

Discussion of Results

Below are the results from Matlab analysis of the collected data:

Response Type	Mean (nF)	Lower 95% Confidence Limit (nF)	Upper 95% Confidence Limit (nF)	Confidence Window
Step	34.755	33.141	36.369	3.228
Magnitude Fit	32.954	32.583	33.325	0.742
Phase Fit	33.335	33.061	33.609	0.548
Combination Fit	33.145	32.902	33.388	0.486

Table 8: Capacitor Mean Values and Associated Confidence Limits from Matlab Script

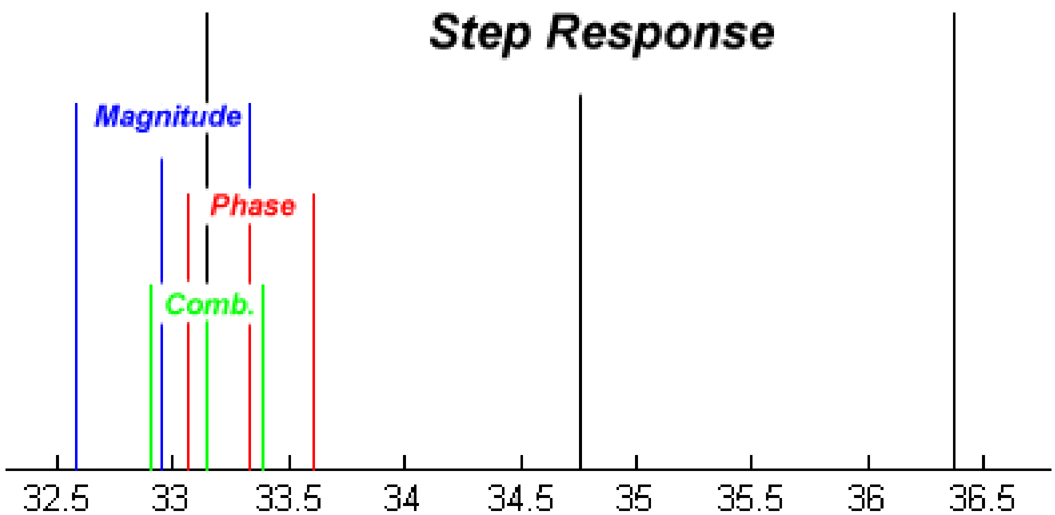

Figure 3: Number Line Representation

Step Response

From these values, it is obvious that the step response measurements had the most variation with, a confidence window four times that of any other part of the experiment. This is not surprising, as using the cursors to perform measurements on the screen of an oscilloscope cannot be nearly as accurate as having a program adjust the frequency generator and record the results, as happened in all the other trials. The step response does not fall within the confidence limits for any other measurement. A possible reason for this could be that different resistor values were used for these measurements, making the internal resistance of the function generator meaningful.

LabView Controlled Response

Using LabView to both control the instruments and collect the data resulted in much smaller variance of data. Looking at the confidence limits, the mean value should be accurate to the hundredths digit. All mean values very nearly fall within each respective confidence interval, and all fall within the larger, step response interval.

It is much more efficient to use automated collection of data than to measure manually, as the accuracy in these trials demonstrates. In addition, accuracy was gained through this method due to the theory behind it – the transfer function for this circuit was independent of the internal resistance of the function generator. Small voltage division effects were present in the step response trial, and could only be made negligible by using large resistance values for our known resistor.

Conclusion

During this experiment, two different methods for calculating the capacitor value of an unknown element were demonstrated. First, a voltage step was applied to a series RC circuit and an oscilloscope was used to measure the time for the capacitor to charge from 0 to 63%, known to be equal to the product of the known resistor value and the unknown capacitor value. During this lab, it was shown that the accuracy of manually measuring a time period using the oscilloscope was the inferior of the two methods.

The frequency response model relied on generating a transfer function, which was invariant to input impedance given that we based our reference across only our known resistor and unknown capacitor. Using the magnitude and phase forms of these equations, which are functions of frequency, it is possible to generate curves based on the point at which the frequency, resistance, and capacitance values cancel. This point, known as the cutoff frequency, was varied throughout a reasonable range to find the curve which best fit the measured data. From this frequency measurement and the known resistance, the capacitor value can be found.

Overall, the confidence limits attest to the reasonability of the data. The capacitor value generated by each trial did not change significantly, and falls within the realm of experimental error.