Robust adaptive control of uncertain non-linear systems with non-linear parameterisation

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Abstract: Many physical systems such as welding processes are typically non-linear and uncertain. To control such systems effectively the control scheme requires to be robust with respect to varying operating conditions. On the basis of robust control Lyapunov function method and adaptive non-linear damping approach, this paper presents a robust adaptive control scheme for uncertain non-linear systems with non-linear parameterisation. With the proposed method, estimation of the unknown parameters of the system and generation of an additional signal are not required. There is only one adaptive parameter no matter how high the order of the system is and how many unknown parameters there are. It is shown that the proposed robust adaptive control scheme guarantees the stability of the closed-loop system in the presence of unknown parameters, disturbances, non-linear uncertainties and unmodelled dynamics. Simulation results illustrate the effectiveness of the proposed robust adaptive controller.

Keywords: welding processes; non-linear systems; non-linear parameterisation; uncertainty; unmodelled dynamics; robust control; adaptive control; Lyapunov stability; backstepping.

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1 Introduction

As many physical systems such as manufacturing systems, power systems and biochemical processes are inherently non-linear systems with uncertainties, research on robust adaptive control of non-linear systems has received great attention (Jiang and Praly, 1998; Krstic et al., 1996; Liu and Li, 2002, 2003, 2004; Lin and Pongvuthithum, 2003; Lin and Qian, 2002a,b; Zhang and Ioannou, 1999). Based on the ideas of adding a power integrator, a few important results have been presented in Lin and Qian (2002a,b) and Lin and Pongvuthithum (2003) for non-linear systems with non-linear parameterisation. However, the adaptive control schemes presented by Lin and Qian (2002a,b) cannot be used in systems with unmodelled dynamics. Although the control scheme given by Lin and Pongvuthithum (2003) can be used in the case with unmodelled dynamics, the adaptive law is of switch type, which may cause undesirable behaviour such as chattering. In Liu et al. (2005), a robust adaptive control scheme is proposed for non-linearly parameterised systems with unmodelled dynamics, but it needs to generate an additional signal to dominate the effects of unmodelled dynamics.

On the other hand, significant efforts have been made to control welding processes with an emphasis to overcome their uncertainties and non-linearities. To deal with the parametric uncertainty owing to the varying welding conditions such as the thickness of the material, an interval model is used to describe the welding process and a predictive control algorithm is proposed in Zhang and Kovacevic (1997). In the control of pulsed gas metal arc welding (Zhang et al., 2002), the operational parameters are considered as unfixed and their ranges are used to quantify the resultant uncertainty in the dynamic model. Then, a single control algorithm is employed at different operational parameters. Experiments show that the control scheme is robust with respect to the variations in wire speed and contact tube-to-work distance. In Zhang and Liu (2003), an adaptive control algorithm is developed for quasi-keyhole arc welding process to achieve the desired peak current duration. On the basis of the analysis of the quasi-keyhole arc welding process, non-linear interval models are obtained by Lu et al. (2004) to control the process. Experiments verified that the non-linear model-based interval model control has advantages over the linear model-based interval model control and the linear model-based adaptive predictive control.

In Thomsen (2005), a non-linear controller based on the feedback linearisation is proposed for arc length control. With the non-linear controller, no operation points need to be selected; therefore, only one controller needs to be tuned for all possible arc lengths and current (or electrode speed) settings. In summary, various advanced control schemes have been presented to overcome uncertainties and non-linearities in welding processes. However, to the best knowledge of the author, no control scheme has been proposed to deal with the unmodelled dynamics in the welding processes.

On the basis of the robust control Lyapunov function method, this paper presents a robust adaptive control scheme for non-linearly parameterised systems with uncertainties. Adaptive non-linear damping is used to restrain the effects of unmodelled dynamics, non-linear uncertainties and disturbances. The backstepping procedure is employed to overcome the complexity in the design of the controller. Estimation of the unknown parameters of the system and generation of an additional signal are not required with the proposed method. No matter how high the order of the system is and how many unknown parameters there are, there is only one adaptive parameter. It is proved theoretically that the proposed robust adaptive control scheme guarantees the stability of the closed-loop system. Simulation results illustrate the effectiveness of the proposed robust adaptive controller.

2 Problem statement

Consider a non-linearly parameterised system

$$\begin{cases} \dot{x}_1 = x_2 + \phi_1(x,\theta,\omega,\mathrm{d}(t)) \\ \ldots\ldots \\ \dot{x}_{n-1} = x_n + \phi_{n-1}(x,\theta,\omega,\mathrm{d}(t)) \\ \dot{x}_n = u + \phi_n(x,\theta,\omega,\mathrm{d}(t)) \end{cases} \tag{1}$$

where $u \in R$ is the control, $x \in R^n$ is the state $\theta \in R^m$ is the unknown parameter, $\mathrm{d}(t) \in R^l$ is the unknown bounded disturbance, $\phi_i(x,\theta,\omega,\mathrm{d}(t))$ represents the uncertain nonlinearity and the uncertainty related to the unmodelled dynamics and disturbance. $\omega \in R^r$ is the unmodelled dynamics described by

$$\dot{\omega} = q\left(\omega, x_1\right) \tag{2}$$

where $q(\omega, x_1)$ is an unknown Lipschitz continuous function.

Without the loss of the generality, it is assumed that the equilibrium of system (1) is 0, and $\phi_i(0,\theta,\omega,d(t)) = 0$, $i = 1, \ldots, n$.

The objective of this paper is to design a robust adaptive controller for system (1) such that the closed-loop system is stable in the presence of unknown parameters, disturbances, non-linear uncertainties and unmodelled dynamics. To this end, the following assumptions are needed.

Assumption 1: *$\phi_i(x,\theta,\omega,d(t)), i = 1, 2, \ldots, n$, are unknown Lipschitz continuous functions satisfying*

$$\left|\phi_i(x,\theta,\omega,d(t))\right| \le c_{i1}\gamma_i\left(x_1,\ldots,x_i\right) + c_{i2}\beta_i\left(\|\omega\|\right) + c_{i3} \tag{3}$$

where $c_{i1}, c_{i2}, c_{i3} \ge 0$ are unknown constants. $\gamma_i(x_1,\ldots, x_i)$, $i = 1,\ldots, n$ are smooth functions and $\beta_i(\| \omega \|)$, $i = 1,\ldots,n$ are k_∞ functions.

Assumption 2: *The unmodelled dynamics described in (2) is Input-to-State Stable (ISS); that is, system (2) has an ISS Lyapunov function $V_\omega(\omega)$ satisfying*

$$\alpha_1\left(\|\omega\|\right) \le V_\omega(\omega) \le \alpha_2\left(\|\omega\|\right) \tag{4}$$

$$\frac{\partial V_\omega(\omega)}{\partial \omega} q\left(\omega, x_1\right) \le -W\left(\|\omega\|\right) + \alpha\left(|x_1|\right) \tag{5}$$

where α_1, α_2, α and W are functions of class k_∞.

On the basis of the idea of changing supply functions (Sontag and Teel, 1995), one can assume

$$V_0(\omega) = \int_0^{V_\omega(\omega)} \eta(s)\mathrm{d}s \tag{6}$$

where $\eta(\cdot) \ge 1$ is a monotone non-decreasing function. From Assumption 2, it can be shown that for any given function $\delta(\omega) \ge 0$, a function $\eta(\cdot) \ge 1$ can be found such that

$$\frac{\partial V_0(\omega)}{\partial \omega} q\left(\omega, x_1\right) \le -\frac{1}{2}\eta\left(V_\omega(\omega)\right)W\left(\|\omega\|\right) + x_1^2\hat{\alpha}\left(|x_1|\right) \tag{7}$$

$$\frac{1}{4\lambda}\eta\left(V_\omega(\omega)\right)W\left(\|\omega\|\right) \ge \delta(\omega) \tag{8}$$

where $\lambda \ge 1$ is a constant; $\hat{\alpha}(|x_1|) \ge 0$ is a known function.

3 Design of robust adaptive controller

In this section, the backstepping design procedure in Kanellakopoulos et al. (1991) is employed to obtain the robust adaptive controllers.

Step 1: Define $\tilde{\Theta} = \Theta^* - \hat{\Theta}(t)$, where $\hat{\Theta}(t)$ is the adaptive parameter of the controller; $\Theta^* > 0$ is an unknown constant representing the desired value of $\hat{\Theta}$, that is, when $\hat{\Theta} = \Theta^*$, system (1) has the desired performance. Choose the Lyapunov function candidate as

$$V_1 = \frac{1}{\lambda}V_0(\omega) + \frac{1}{2}\xi_1^2 + \frac{1}{2}\Gamma^{-1}\tilde{\Theta}^2 \tag{9}$$

where $\xi_1 = x_1$, $\Gamma > 0$ is a constant. Then,

$$\begin{aligned} \dot{V}_1 \le & -\frac{1}{2\lambda}\eta\left(V_\omega(\omega)\right)W\left(\|\omega\|\right) + \frac{1}{\lambda}\xi_1^2\hat{\alpha}\left(\xi_1\right) \\ & + \xi_1 x_2 + c_{11}\left|\xi_1\right|\gamma_1\left(x_1\right) + c_{12}\left|\xi_1\right|\beta_1\left(\|\omega\|\right) \\ & + c_{13}\left|\xi_1\right| + \left|\xi_1\right|\left|\dot{y}_r\right| - \Gamma^{-1}\tilde{\Theta}\dot{\hat{\Theta}} \end{aligned} \tag{10}$$

Let $\tilde{c}_{12} = 1/2\, c_{12}^2$, $\tilde{c}_{13} = M + c_{13}$.

As a result,

$$\begin{aligned}\dot{V}_1 \le & -\frac{1}{2\lambda}\eta\left(V_\omega(\omega)\right)W\left(\|\omega\|\right)+\frac{1}{\lambda}\xi_1^2\hat{\alpha}\left(\xi_1\right)+\xi_1 x_2 \\ & -\Theta^*\left(\left|\xi_1\right|\gamma_1\left(x_1\right)-\frac{c_{11}}{2\Theta^*}\right)^2-\Theta^*\left(\xi_1^2-\frac{\tilde{c}_{12}}{2\Theta^*}\right)^2 \\ & -\Theta^*\left(\left|\xi_1\right|-\frac{\tilde{c}_{13}}{2\Theta^*}\right)^2+\frac{c_{11}^2}{4\Theta^*}+\frac{\tilde{c}_{12}}{4\Theta^*}+\frac{\tilde{c}_{13}^2}{4\Theta^*} \\ & +\left(\xi_1^2\gamma_1^2\left(x_1\right)+\xi_1^4+\xi_1^2-\sigma\hat{\Theta}-\Gamma^{-1}\dot{\hat{\Theta}}\right)\left(\Theta^*-\hat{\Theta}\right) \\ & +\hat{\Theta}\left(\xi_1^2\gamma_1^2\left(x_1\right)+\xi_1^4+\xi_1^2\right)+\sigma\hat{\Theta}\left(\Theta^*-\hat{\Theta}\right) \\ & +\frac{1}{2}\beta_1^2\left(\|\omega\|\right)\end{aligned} \tag{11}$$

Choose virtual control

$$\begin{aligned}x_2^*\left(x_1,y_r,\hat{\Theta}\right)= & -\frac{1}{\lambda}\xi_1\hat{\alpha}\left(\xi_1\right)-\frac{3}{2}\xi_1 \\ & -\hat{\Theta}\xi_1\left[\gamma_1^2\left(x_1\right)+\xi_1^2+1\right]\end{aligned} \tag{12}$$

Then,

$$\begin{aligned}\dot{V}_1 \le & -\frac{1}{2\lambda}\eta\left(V_\omega(\omega)\right)W\left(\|\omega\|\right)-\xi_1^2 \\ & -\frac{1}{2}\xi_1^2+\xi_1\left(x_2-x_2^*\right) \\ & +\left(\psi_1\left(x_1,y_r,\hat{\Theta},s\right)-\Gamma^{-1}\dot{\hat{\Theta}}(t)\right)\left(\tilde{\Theta}(t)+\varphi_1\right) \\ & +N_1+\sigma\hat{\Theta}\tilde{\Theta}+\frac{1}{2}\beta_1^2\left(\|\omega\|\right)\end{aligned} \tag{13}$$

where

$$N_1=\frac{c_{11}^2}{4\Theta^*}+\frac{\tilde{c}_{12}^2}{4\Theta^*}+\frac{\tilde{c}_{13}^2}{4\Theta^*},\quad \varphi_1=0$$

$$\psi_1\left(x_1,\hat{\Theta}\right)=\xi_1^2\gamma_1^2\left(x_1\right)+\xi_1^4+\xi_1^2-\sigma\hat{\Theta} \tag{14}$$

$\sigma>0$ is a design constant.

Step k ($2 \le k \le n-1$): Assume a series of virtual controllers had been developed before step k

$$x_2^*=x_2^*\left(x_1,\hat{\Theta}\right),\ldots,\qquad x_{k+1}^*=x_{k+1}^*\left(x_1,\ldots,x_k,\hat{\Theta}\right)$$

Define

$$\begin{cases}\xi_1=x_1 \\ \xi_2=x_2-x_2^* \\ \ldots\ldots \\ \xi_{k+1}=x_{k+1}-x_{k+1}^*\end{cases} \tag{15}$$

In step k, the Lyapunov function candidate is

$$V_k=\frac{1}{\lambda}V_0(\omega)+\sum_{j=1}^{k}\frac{1}{2}\xi_j^2+\frac{1}{2}\Gamma^{-1}\tilde{\Theta}^2 \tag{16}$$

It is also assumed that in step k, similar to step 1, by choosing an appropriate virtual controller $x_{k+1}^*(x_1,\ldots,x_k,\hat{\Theta})$, one can obtain

$$\begin{aligned}\dot{V}_k \le & -\frac{1}{2\lambda}\eta\left(V_\omega(\omega)\right)W\left(\|\omega\|\right)-\left(\xi_1^2+\cdots+\xi_k^2\right) \\ & -\frac{1}{2}\xi_k^2+\xi_k\left(x_{k+1}-x_{k+1}^*\right) \\ & +\frac{1}{2}\sum_{i=1}^{k}(k-i+1)\beta_i^2\left(\|\omega\|\right) \\ & +\left(\psi_k\left(x_1,\ldots,x_k,\hat{\Theta}\right)-\Gamma^{-1}\dot{\hat{\Theta}}\right)\left(\tilde{\Theta}+\varphi_k\right) \\ & +N_k+\sigma\hat{\Theta}\tilde{\Theta}\end{aligned} \tag{17}$$

where

$$\begin{aligned}N_k &= N_{k-1}+\frac{k\tilde{c}_{k,1}^2}{4\Theta^*}+\frac{k\tilde{c}_{k,2}^2}{4\Theta^*}+\frac{(k+1)\tilde{c}_{k,3}^2}{4\Theta^*} \\ k &= 2,3,\ldots, \\ N_0 &= 0,\ N_1=\frac{c_{11}^2}{4\Theta^*}+\frac{\tilde{c}_{12}^2}{4\Theta^*}+\frac{\tilde{c}_{13}^2}{4\Theta^*}\end{aligned}$$

$\tilde{c}_{k,1},\ \tilde{c}_{k,2},\ \tilde{c}_{k,3}\ge 0$, are unknown constants and $\tilde{c}_{11}=c_{1,1}$. In the following, the author proves that in step k+1, by choosing virtual controller $x_{k+2}^*(x_1,\ldots,x_{k+1},\hat{\Theta})$, (17) also holds. To this end, define

$$V_{k+1}=V_k+\frac{1}{2}\xi_{k+1}^2 \tag{18}$$

Then,

$$\begin{aligned}\dot{V}_{k+1} \le & -\frac{1}{2\lambda}\eta\left(V_\omega(\omega)\right)W\left(\|\omega\|\right) \\ & -\left(\xi_1^2+\cdots+\xi_k^2\right)-\frac{1}{2}\xi_k^2+\xi_k\left(x_{k+1}-x_{k+1}^*\right) \\ & +\left(\psi_k\left(x_1,\ldots,x_k,\hat{\Theta}\right)-\Gamma^{-1}\dot{\hat{\Theta}}\right)\left(\tilde{\Theta}+\varphi_k\right) \\ & +\frac{1}{2}\sum_{i=1}^{k}(k-i+1)\beta_i^2\left(\|\omega\|\right) \\ & +\xi_{k+1}\left(x_{k+2}+\phi_{k+1}(x,\theta,\omega,d(t))\right) \\ & -\xi_{k+1}\left(\sum_{j=1}^{k}\frac{\partial x_{k+1}^*}{\partial x_j}\left(x_{j+1}+\phi_j(\cdot)\right)+\frac{\partial x_{k+1}^*}{\partial\hat{\Theta}}\dot{\hat{\Theta}}\right) \\ & +N_k+\sigma\,\hat{\Theta}\tilde{\Theta}\end{aligned} \tag{19}$$

As

$$\begin{aligned} & \left|\xi_{k+1}\right|\left|\phi_{k+1}(\cdot)-\sum_{j=1}^{k}\frac{\partial x_{k+1}^*}{\partial x_j}\phi_j(\cdot)\right| \\ & \le \tilde{c}_{k+1,1}\left|\xi_{k+1}\right|\left(\gamma_{k+1}(\cdot)+\sum_{j=1}^{k}\left|\frac{\partial x_{k+1}^*}{\partial x_j}\right|\gamma_j(\cdot)\right) \\ & +\tilde{c}_{k+1,2}\left|\xi_{k+1}\right|\left(\beta_{k+1}\left(\|\omega\|\right)+\sum_{j=1}^{k}\left|\frac{\partial x_{k+1}^*}{\partial x_j}\right|\beta_j\left(\|\omega\|\right)\right) \\ & +\hat{c}_{k+1,3}\left|\xi_{k+1}\right|\left(1+\sum_{j=1}^{k}\left|\frac{\partial x_{k+1}^*}{\partial x_j}\right|\right)\end{aligned} \tag{20}$$

where

$$\tilde{c}_{k+1,1}=\max\left\{c_{1,1},\ldots,c_{k+1,1}\right\}$$

$$\tilde{c}_{k+1,2}=\max\left\{c_{1,2},\ldots,c_{k+1,2}\right\}$$

$$\tilde{c}_{k+1,3} = \max\left(c_{1,3},\ldots,c_{k+1,3}\right)$$

and

$$\left|\xi_k\left(x_{k+1} - x^*_{k+1}\right)\right| \le \frac{1}{2}\xi_k^2 + \frac{1}{2}\xi_{k+1}^2$$

The following can be obtained:

$$\begin{aligned}\dot{V}_{k+1} \le & -\frac{1}{2\lambda}\eta\left(V_\omega(\omega)\right)W\left(\|\omega\|\right) - \left(\xi_1^2 + \cdots + \xi_k^2\right) \\ & + \frac{1}{2}\xi_{k+1}^2 + \left(\psi_k\left(x_1,\ldots,x_k,\hat{\Theta}\right) - \Gamma^{-1}\dot{\hat{\Theta}}\right)\left(\tilde{\Theta} + \varphi_k\right) \\ & + \frac{1}{2}\sum_{i=1}^{k}(k-i+1)\beta_i^2\left(\|\omega\|\right) + \xi_{k+1}x_{k+2} \\ & - \xi_{k+1}\left(\sum_{j=1}^{k}\frac{\partial x^*_{k+1}}{\partial x_j}x_{j+1} + \frac{\partial x^*_{k+1}}{\partial\hat{\Theta}}\dot{\hat{\Theta}}\right) + N_k + \sigma\,\hat{\Theta}\tilde{\Theta} \\ & + \tilde{c}_{k+1,1}\left|\xi_{k+1}\right|\left(\gamma_{k+1}(\cdot) + \sum_{j=1}^{k}\left|\frac{\partial x^*_{k+1}}{\partial x_j}\right|\gamma_j(\cdot)\right) \\ & + \tilde{c}_{k+1,2}\left|\xi_{k+1}\right|\left(\beta_{k+1}\left(\|\omega\|\right) + \sum_{j=1}^{k}\left|\frac{\partial x^*_{k+1}}{\partial x_j}\right|\beta_j\left(\|\omega\|\right)\right) \\ & + \tilde{c}_{k+1,3}\left|\xi_{k+1}\right|\left(1 + \sum_{j=1}^{k}\left|\frac{\partial x^*_{k+1}}{\partial x_j}\right|\right)\end{aligned} \tag{21}$$

Denote

$$\begin{aligned}\psi_{k+1} = & \psi_k(\cdot) + \xi_{k+1}^2\left(1 + \sum_{j=1}^{k}\left(\frac{\partial x^*_{k+1}}{\partial x_j}\right)^2\right) \\ & + \xi_{k+1}^2\left(\gamma_{k+1}^2(\cdot) + \sum_{j=1}^{k}\left(\frac{\partial x^*_{k+1}}{\partial x_j}\gamma_j(\cdot)\right)^2\right)\end{aligned} \tag{22}$$

$$\varphi_{k+1} = \varphi_k + \Gamma\xi_{k+1}\frac{\partial x^*_{k+1}}{\partial\hat{\Theta}} \tag{23}$$

Choose virtual controller

$$\begin{aligned}x^*_{k+2} = & -2\xi_{k+1} + \sum_{j=1}^{k}\frac{\partial x^*_{k+1}}{\partial x_j}x_{j+1} + \Gamma\frac{\partial x^*_{k+1}}{\partial\hat{\Theta}}\psi_{k+1} \\ & - \hat{\Theta}\xi_{k+1}\left(1 + \sum_{j=1}^{k}\left(\frac{\partial x^*_{k+1}}{\partial x_j}\right)^2\right) \\ & - \hat{\Theta}\xi_{k+1}\left(\gamma_{k+1}^2(\cdot) + \sum_{j=1}^{k}\left(\frac{\partial x^*_{k+1}}{\partial x_j}\gamma_j(\cdot)\right)^2\right)\end{aligned} \tag{24}$$

Then, from (21) one can obtain

$$\begin{aligned}\dot{V}_{k+1} \le & -\frac{1}{2\lambda}\eta\left(V_\omega(\omega)\right)W\left(\|\omega\|\right) - \left(\xi_1^2 + \cdots + \xi_{k+1}^2\right) \\ & - \frac{1}{2}\xi_{k+1}^2 + \xi_{k+1}\left(x_{k+2} - x^*_{k+2}\right) \\ & + \left(\psi_{k+1}\left(x_1,\ldots,x_{k+1},\hat{\Theta},s\right) - \Gamma^{-1}\dot{\hat{\Theta}}\right)\left(\tilde{\Theta} + \varphi_{k+1}\right) \\ & + \frac{1}{2}\sum_{i=1}^{k+1}(k+1-i+1)\beta_i^2\left(\|\omega\|\right) + N_{k+1} + \sigma\hat{\Theta}\tilde{\Theta}\end{aligned} \tag{25}$$

where

$$N_{k+1} = \frac{(k+1)\tilde{c}_{k+1,1}^2}{4\Theta^*} + \frac{(k+1)\tilde{c}_{k+1,2}^2}{4\Theta^*} + \frac{(k+2)\tilde{c}_{k+1,3}^2}{4\Theta^*} + N_k$$

Hence, it has been proved that in step *K*+1, by choosing virtual controller $x^*_{k+2}(x_1,\ldots,x_{k+1},\hat{\Theta})$ as in (24), (17) also holds.

Step n: It can be seen from (1) that Equations (22)–(24) also hold in step *n* provided that x_{n+1} is replaced by u. Take

$$V_n = \sum_{j=1}^{n}\frac{1}{2}\xi_j^2 + \frac{1}{2}\Gamma^{-1}\tilde{\Theta}^2 + \frac{1}{\lambda}V_0(\omega) \tag{26}$$

and

$$u = u^*\left(x_1,\ldots,x_n,\hat{\Theta}\right) = x^*_n\left(x_1,\ldots,x_n,\hat{\Theta}\right) \tag{27}$$

One can get

$$\begin{aligned}\dot{V}_n \le & -\frac{1}{2\lambda}\eta\left(V_\omega(\omega)\right)W\left(\|\omega\|\right) - \left(\xi_1^2 + \cdots + \xi_n^2\right) \\ & - \frac{1}{2}\xi_n^2 + \xi_n\left(u - u^*\right) \\ & + \left(\psi_n(\cdot) - \Gamma^{-1}\dot{\hat{\Theta}}\right)\left(\tilde{\Theta} + \varphi_n\right) \\ & + \frac{1}{2}\sum_{i=1}^{n}(n-i+1)\beta_i^2\left(\|\omega\|\right) + N_n + \sigma\hat{\Theta}\tilde{\Theta}\end{aligned} \tag{28}$$

Let $\delta(\omega) = \frac{1}{2}\sum_{i=1}^{n}(n-i+1)\beta_i^2\left(\|\omega\|\right)$. From (8),

$$\begin{aligned}\dot{V}_n \le & -\frac{1}{4\lambda}\eta\left(V_\omega(\omega)\right)W\left(\|\omega\|\right) - \left(\xi_1^2 + \cdots + \xi_n^2\right) \\ & - \frac{1}{2}\xi_n^2 + \xi_n\left(u - u^*\right) \\ & + \left(\psi_n(\cdot) - \Gamma^{-1}\dot{\hat{\Theta}}\right)\left(\tilde{\Theta} + \varphi_n\right) \\ & + N_n + \sigma\,\hat{\Theta}\tilde{\Theta}\end{aligned}$$

Choosing the adaptive law

$$\dot{\hat{\Theta}} = \Gamma\psi_n\left(x_1,\ldots,x_n,\hat{\Theta}\right), \qquad \hat{\Theta}(0) > 0 \tag{29}$$

gives

$$\begin{aligned}\dot{V}_n\left(\xi_1,\ldots,\xi_n,\tilde{\Theta}\right) \le & -\frac{1}{4\lambda}\eta\left(V_\omega(\omega)\right)W\left(\|\omega\|\right) \\ & - \left(\xi_1^2 + \cdots + \xi_n^2\right) + N_n + \sigma\,\hat{\Theta}\tilde{\Theta}\end{aligned} \tag{30}$$

where

$$N_n = \sum_{k=1}^{n}\frac{k\tilde{c}_{k,1}^2}{4\Theta^*} + \frac{k\tilde{c}_{k,2}^2}{4\Theta^*} + \frac{(k+1)\tilde{c}_{k,3}^2}{4\Theta^*}$$

Note that the $x^*_n(x_1,\ldots,x_n,\hat{\Theta})$ in (27) and $\psi_n(x_1,\ldots,x_n,\hat{\Theta})$ in (29) satisfy (24) and (22), respectively.

As $\eta(\cdot) \geq 1$ and $\tilde{\Theta} = \Theta^* - \hat{\Theta}$, from (30) the following can be obtained:

$$\dot{V}_n \leq -\frac{1}{4\lambda} W\left(\|\omega\|\right) - \left(\xi_1^2 + \cdots + \xi_n^2\right) - \frac{1}{2}\sigma\tilde{\Theta}^2 + N \tag{31}$$

where

$$N = \frac{1}{2}\sigma\,\Theta^{*2} + \sum_{k=1}^{n} \frac{\tilde{c}_{k,1}^2}{4\Theta^*} + \frac{\tilde{c}_{k,2}^2}{4\Theta^*} + \frac{\tilde{c}_{k,3}^2}{4\Theta^*} \tag{32}$$

Thus, the closed-loop system is stable and all the variables of the closed-loop system are bounded.

4 An illustration example

Consider a non-linearly parameterised system

$$\begin{aligned} \dot{x}_1 &= x_2 + \theta_1 x_1^2 + \theta_2 \sin(x_1)\omega + \theta_3 x_1 d_1(t) \\ \dot{x}_2 &= u + \theta_4 x_1 + \frac{\theta_5 x_2^2}{1 + \theta_6 x_1^2} \\ &\quad + \theta_7\left(1 - \cos\left(x_2\right)\right)\omega + \theta_8 x_1 d_2(t) \end{aligned} \tag{33}$$

where θ_i, $i = 0, 1, \ldots, 8,$ are unknown parameters, ω is the unmodelled dynamics described by

$$\dot{\omega} = -\omega + x_1^2 \tag{34}$$

Obviously,

$$V_\omega(\omega) = \omega^2, \quad \hat{\alpha}\left(\xi_1\right) = 16\xi_1^2$$

According to the proposed control scheme,

$$\gamma_1\left(x_1\right) = x_1^2, \quad \gamma_2\left(x_1, x_2\right) = x_1^2 + x_2^2$$

$$\psi_1\left(x_1, \hat{\Theta}\right) = \xi_1^2 x_1^4 + \xi_1^4 + \xi_1^2 - \sigma\hat{\Theta}$$

The virtual controller

$$x_2^*\left(x_1, \hat{\Theta}\right) = -\frac{16}{\lambda}\xi_1^3 - \frac{3}{2}\xi_1 - \hat{\Theta}\xi_1\left[x_1^4 + \xi_1^2 + 1\right]$$

$$\psi_2 = \psi_1(\cdot) + \xi_2^2\left(1 + \left(\frac{\partial x_2^*}{\partial x_1}\right)^2\right) + \xi_2^2\left(\gamma_2^2(\cdot) + \left(\frac{\partial x_2^*}{\partial x_1}\gamma_1(\cdot)\right)^2\right)$$

The adaptive law for the controller parameter is

$$\dot{\hat{\Theta}} = \Gamma\psi_2(\cdot)$$

The robust adaptive controller is

$$\begin{aligned} u = u^* = x_3^* &= -2\xi_2 + \frac{\partial x_2^*}{\partial x_1} x_2 \\ &\quad + \Gamma\frac{\partial x_2^*}{\partial \hat{\Theta}}\psi_2 - \hat{\Theta}\xi_2\left(1 + \left(\frac{\partial x_2^*}{\partial x_1}\right)^2\right) \\ &\quad - \hat{\Theta}\xi_2\left(\gamma_2^2(\cdot) + \left(\frac{\partial x_2^*}{\partial x_1}\gamma_1(\cdot)\right)^2\right) \end{aligned}$$

In the simulation,

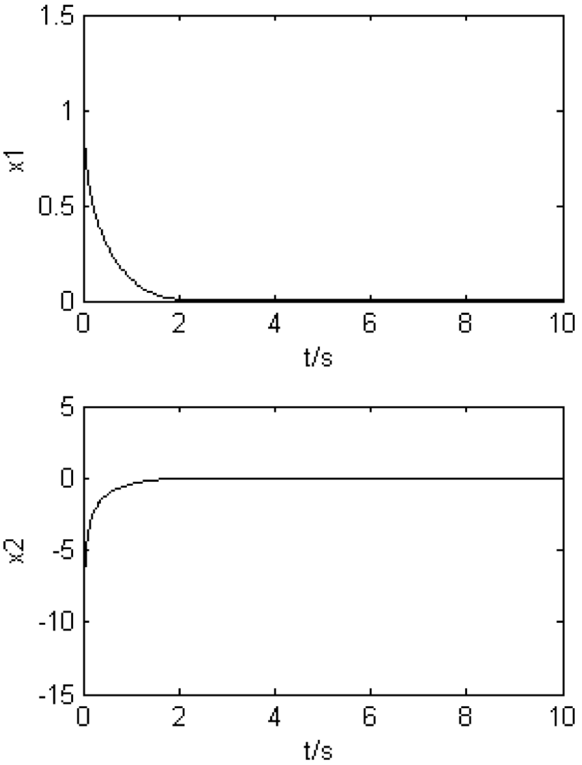
$$\begin{aligned} &d_1(t) = \sin(t),\ d_2(t) = \cos(t) \\ &\theta_1 = 2,\ \theta_2 = 2,\ \theta_3 = 0.5,\ \theta_4 = 5, \\ &\theta_5 = 6,\ \theta_6 = 2,\ \theta_7 = 1,\ \theta_8 = 0.5 \\ &x_1(0) = 1,\ x_2(0) = 1,\ \omega(0) = 1,\ \hat{\Theta}(0) = 1 \end{aligned}$$

The design constants are

$$\lambda = 10,\ \Gamma = 1,\ \sigma = 0.001$$

It can be seen from the simulation results shown in Figure 1 that the state of the system approaches the equilibrium and all variables of the closed-loop system are bounded in the presence of unmodelled dynamics and bounded disturbances.

Figure 1 Simulation results



5 Conclusion

A robust adaptive control scheme is presented for non-linearly parameterised systems with unknown parameters, uncertain non-linearities, disturbances and unmodelled dynamics. The scheme does not need to estimate the unknown parameters nor to add a dynamical signal. No matter how high the order of the system is and how many unknown parameters the system has, there is only one adaptive parameter. It is proved theoretically that the proposed robust adaptive control scheme guarantees the stability of the closed-loop system. Simulation results illustrated the effectiveness of the robust adaptive controller. Further work will be to use the proposed control scheme in some practical uncertain non-linear systems such as the welding processes.

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