

# Robust adaptive control of nonlinear systems with unmodelled dynamics

Y. Liu and X.-Y. Li

**Abstract:** A robust adaptive output feedback controller is proposed for a class of nonlinear systems represented by input–output models. In the design of the adaptive controller, a signal is used to characterise the unmodelled dynamics and a nonlinear damping term is introduced to counteract the effects of the unmodelled dynamics and bounded disturbances. With the proposed controller, all the variables in the closed-loop system are bounded in the presence of unmodelled dynamics and bounded disturbances. Moreover, the mean-square tracking error can be made arbitrarily small by choosing appropriate design parameters.

## 1 Introduction

In the past decade, various adaptive control schemes have been proposed for nonlinear systems. Particularly, an adaptive output feedback controller for a class of nonlinear systems represented by input–output models has been presented in [1]. The adaptive controller in [1] has several advantages: it can be applied to a wide class of nonlinear systems; the models of the systems may depend nonlinearly on control input  $u$ ; and, it is simpler than the traditional adaptive output feedback controllers in that filtering or error augmentation is not required. However, the adaptive controller is only robust to sufficiently fast unmodelled dynamics and sufficiently small bounded disturbances.

Robust adaptive control of nonlinear systems has emerged as an active research area [2, 3]. By adding a robustifying control component, the results of [1] have been extended to the case with nonsmall disturbance in [4], but an upper bound on the disturbance must be known in the design of the controller. While many results in the field of robust adaptive control mainly deal with parameter uncertainty and uncertain nonlinearities, robust control for nonlinear systems with unmodelled dynamics has received considerable attention [5–10]. But, few results are available about the robust adaptive control of the nonlinear systems represented by input–output models with unmodelled dynamics. Using the approach of [6], the authors of [11] extended their previous work to the case of unmodelled dynamics. As in [4], the robustifying control component employed in [11] was a type of switch controller with a switch parameter  $\mu_v$ . The mean-square tracking error in [11] is of order of  $O(\mu_v + \varepsilon)$ , i.e. the tracking error depends on the switch parameter  $\mu_v$  in addition to the small constant  $\varepsilon$ . Although the mean-square tracking error can theoretically be made arbitrarily small by choosing sufficiently small

$\mu_v$  and  $\varepsilon$ , too small a switch parameter may cause undesirable behaviour such as chattering.

Using a nonlinear damping term to counteract the effects of the unmodelled dynamics and bounded disturbances, this paper presents a robust adaptive output feedback controller for a class of nonlinear systems represented by input–output models containing unmodelled dynamics and bounded time-varying disturbances. In the design of the adaptive controller, it is not necessary to know the upper bound of the disturbances. When the derivatives of the output are available for feedback, the adaptive controller guarantees that all the variables of the closed-loop system are bounded; moreover, the mean-square tracking error can be made arbitrarily small by choosing an appropriate design parameter. To implement the robust adaptive controller via output feedback, we use a high-gain observer to estimate the tracking errors. In addition to saturating the control, we also saturate some signals outside the region of interest to prevent the peaking phenomenon from entering the control system. With the adaptive output feedback controller, the closed-loop system is stable and the mean-square tracking error is of the order  $O(\varepsilon)$ .

## 2 Problem statement

We consider a single-input-single-output nonlinear system described by

$$\begin{aligned} y^{(n)} &= f_0(y, \dot{y}, \dots, y^{(n-1)}, u, \dot{u}, \dots, u^{(m-1)}) \\ &+ \sum_{i=1}^p f_i(y, \dot{y}, \dots, y^{(n-1)}, u, \dot{u}, \dots, u^{(m-1)}) \theta_i \\ &+ \left( g_0 + \sum_{i=1}^p g_i \theta_i \right) u^{(m)} + \Delta(y, \dot{y}, \dots, y^{(n-1)}, \\ &u, \dot{u}, \dots, u^{(m-1)}, \omega) + d(t) \end{aligned} \quad (1)$$

where  $y$  is the output;  $u$  is the control;  $y^{(i)}$  is the  $i$ th derivative of  $y$ ;  $d(t)$  is the unknown bounded disturbance;  $\Delta(\cdot)$  represents the uncertain nonlinearities and the unmodelled dynamics described later;  $g_i$ ,  $i = 0, 1, \dots, p$  are known constants;  $\theta_i$ ,  $i = 1, \dots, p$ , are unknown parameters; and  $f_i$ ,  $i = 0, 1, \dots, p$  are known smooth nonlinear functions.

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IEE Proceedings online no. 20040063

doi: 10.1049/ip-cta:20040063

Paper first received 10th February and in revised form 23rd September 2003

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Let  $x_1 = y$ ,  $x_2 = y^{(1)}$ ,  $\dots$ ,  $x_n = y^{(n-1)}$ ,  $z_1 = u$ ,  $z_2 = u^{(1)}$ ,  $\dots$ ,  $z_m = u^{(m-1)}$ . By adding a series of  $m$  integrators at the input side of system (1) as in [1], we obtain the following state equations:

$$\begin{aligned} \dot{x}_i &= x_{i+1}, \quad 1 \leq i \leq n-1 \\ \dot{x}_n &= f_0(x, z) + \theta^T f(x, z) + (g_0 + \theta^T g)v + \Delta(x, z, \omega) + d(t) \\ \dot{z}_i &= z_{i+1}, \quad 1 \leq i \leq m-1 \\ \dot{z}_m &= v \end{aligned} \quad (2)$$

where  $v = u^{(m)}$  is the control input for the augmented system (2), and  $x = [x_1, \dots, x_n]^T$ ,  $z = [z_1, \dots, z_m]^T$ ,  $\theta = [\theta_1, \dots, \theta_p]^T$ ,  $g = [g_1, \dots, g_p]^T$ ,  $f = [f_1, \dots, f_p]^T$ . The  $\omega \in R^l$  in  $\Delta(x, z, \omega)$  is the unmodelled dynamics described by

$$\dot{\omega} = q(\omega, x, z) \quad (3)$$

We assume that  $\Delta$  and  $q$  are unknown nonlinear functions, which are continuous and locally Lipschitz over the domain of interests and satisfy

$$|\Delta(x, z, \omega)| \leq c_1 \|x\| + c_2 \|z\| + c_3 \|\omega\| \quad (4)$$

$$\Delta(0, z, 0) = 0 \quad (5)$$

where  $c_1, c_2, c_3 \geq 0$  are unknown constants. In (4) and throughout this paper, we use the Euclidean norm. Let

$$\zeta_i = z_i - \frac{x_{n-m+i}}{g_0 + \theta^T g}, \quad 1 \leq i \leq m \quad (6)$$

From (2), we have

$$\begin{aligned} \dot{\zeta}_i &= \zeta_{i+1}, \quad 1 \leq i \leq m-1 \\ \dot{\zeta}_m &= -\frac{f_0(x, z) + \theta^T f(x, z) + \Delta(x, z, \omega) + d(t)}{g_0 + \theta^T g} \Big|_{z_i = \zeta_i + \frac{x_{n-m+i}}{g_0 + \theta^T g}} \end{aligned} \quad (7)$$

Let

$$\begin{aligned} \zeta &= [\zeta_1, \dots, \zeta_m]^T, \quad \bar{b} = [0, \dots, 0, 1]^T, \quad h(\zeta, x, \theta) \\ &= \left[ \zeta_2, \dots, \zeta_{m-1}, -\frac{f_0(x, z) + \theta^T f(x, z)}{g_0 + \theta^T g} \right]^T \end{aligned}$$

Then, (7) can be written as

$$\dot{\zeta} = h(\zeta, x, \theta) + \bar{b} \left( \frac{-\Delta(x, z, \omega) - d(t)}{g_0 + \theta^T g} \right) \quad (8)$$

Thus, the nominal system of (2) consists of the first  $n$  equations in (2), and (8) with  $\Delta(x, z, \omega) = 0$ ,  $d(t) = 0$ .

We further assume that the reference signal  $y_r(t)$  is bounded with bounded derivatives up to the  $n$ th order and  $y_r^{(n)}(t)$  is piecewise continuous. Denote  $\bar{y}_r = (y_r, y_r^{(1)}, \dots, y_r^{(n-1)})^T$ ,  $\bar{y}_R = (y_r, y_r^{(1)}, \dots, y_r^{(n)})^T$ . Let  $Y \subset R^n$ ,  $Y_R \subset R^{n+1}$ ,  $Z_0 \subset R^m$ ,  $W_0 \subset R^l$  be any given compact sets. Then, the objective of this paper is to design a robust adaptive output feedback controller for system (2) and (3) such that for any  $x(0) \in Y$ ,  $z(0) \in Z_0$ ,  $\omega(0) \in W_0$  and  $\bar{y}_r \in Y_R$ , the output  $y(t)$  of the system tracks the reference signal  $y_r(t)$  and all the variables of the closed-loop system are bounded in the presence of

unmodelled dynamics and bounded disturbances. We need the following assumptions.

*Assumption 1:*  $\theta \in \Omega$ , where  $\Omega$  is a known compact convex subset of  $R^p$ .

*Assumption 2:*  $(g_0 + \theta^T g) \neq 0, \forall \theta \in \hat{\Omega}$ , where  $\hat{\Omega}$  is a convex subset of  $R^p$  and  $\Omega \subset \hat{\Omega}$ .

*Assumption 3:* In the nominal system, the subsystem  $\dot{\zeta} = h(\zeta, x, \theta)$  has a unique steady-state solution  $\bar{\zeta}$ . Without loss of generality, we assume that  $\bar{\zeta} = 0$ . Moreover, the subsystem has a function  $w(t, \zeta)$  satisfying

$$\begin{aligned} \pi_1 \|\zeta\|^2 &\leq w(t, \zeta) \pi_2 \|\zeta\|^2 \\ \frac{\partial w}{\partial t} + \frac{\partial w}{\partial \zeta} h(\zeta, x, \theta) &\leq -\pi_3 \|\zeta\|^2 + \pi_4 \|\zeta\| \|x\| \\ \left\| \frac{\partial w}{\partial \zeta} \right\| &\leq \pi_5 \|\zeta\| \end{aligned} \quad (9)$$

where  $\pi_i > 0, i = 1, \dots, 5$ , are constants and  $\pi_3 > \pi_5 c_2$ .

*Assumption 4:* The unmodelled dynamics, i.e. the system of (3), is exponentially input-to-state practically stable (exp-ISpS) [6]; that is, system (3) has an exp-ISpS Lyapunov function  $V_\omega(\omega)$  satisfying

$$\alpha_1(\|\omega\|) \leq V_\omega(\omega) \leq \alpha_2(\|\omega\|) \quad (10)$$

$$\frac{\partial V_\omega(\omega)}{\partial \omega} q(\omega, x, z) \leq -c_0 V_\omega(\omega) + \gamma(\|x\|) + d_0 \quad (11)$$

where  $\alpha_1, \alpha_2$  are functions of class  $K_\infty$  and  $c_0 > 0, d_0 \geq 0$  are constants. Without loss of generality, we assume that  $\gamma(\cdot)$  has the form  $\gamma(s) = s^2 \gamma_0(s^2)$ , where  $\gamma_0$  is a nonnegative smooth function. Otherwise, as indicated in [6], it suffices to replace  $\gamma$  in (11) by  $\|x\|^2 \gamma_0(\|x\|^2) + \bar{\epsilon}_0$  with  $\bar{\epsilon}_0 > 0$  being a sufficiently small real number.

### 3 Robust adaptive control via state feedback

Let  $e_1 = x_1 - y_r$ ,  $e_2 = x_2 - \dot{y}_r, \dots, e_n = x_n - y_r^{(n-1)}$ ,  $e = [e_1, e_2, \dots, e_n]^T$ . From (2) and (3), we obtain

$$\begin{aligned} \dot{e} &= Ae + b(f_0(e + \bar{y}_r, z) + \theta^T f(e + \bar{y}_r, z) \\ &\quad + (g_0 + \theta^T g)v - y_r^{(n)} + \Delta(e + \bar{y}_r, z, \omega) + d(t)) \\ \dot{z} &= \bar{A}z + \bar{b}v \\ \dot{\omega} &= q(\omega, e + \bar{y}_r, z) \end{aligned} \quad (12)$$

where

$$A = \begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & 1 \\ 0 & 0 & \dots & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

and  $\bar{A}, \bar{b}$  have the same forms as  $A, b$  but with different sizes. Let  $A_m = A - bK$ ,  $\bar{A}_m = \bar{A} - \bar{b}\bar{K}$ , where  $K$  and  $\bar{K}$  are chosen so that  $A_m$  and  $\bar{A}_m$  are Hurwitz polynomials. Then, we have

$$\begin{aligned} \dot{e} &= A_m e + b[Ke + f_0(e + \bar{y}_r, z) + \theta^T f(e + \bar{y}_r, z) \\ &\quad - y_r^{(n)} + (g_0 + \theta^T g)v + \Delta(e + \bar{y}_r, z, \omega) + d(t)] \end{aligned} \quad (13)$$

To characterise the effects of the unmodelled dynamics to the control system, we use a dynamic signal described by

$$\dot{r} = -\bar{c}_0 r + r_m, \quad r(0) = r^0 > 0 \quad (14)$$

where  $\bar{c}_0 \in (0, c_0)$  and  $r_m = \|e + \bar{y}_r\|^2 \gamma_0 (\|e + \bar{y}_r\|^2) + d_0 \triangleq r_m(e, \bar{y}_r)$ . It has been shown in [6] that the signal has the following property:

$$V_\omega(\omega(t)) \leq r(t) + D(t) \quad (15)$$

for all  $t \geq 0$  where the solutions are defined, with  $D(t)$  defined for  $t \geq 0$  and there is a finite  $T^0$  such that  $D(t) = 0$  for all  $t \geq T^0 \geq 0$ .

We propose the following robust adaptive controller:

$$v = \frac{1}{g_0 + \hat{\theta}^T g} \left\{ -Ke + y_r^{(n)} - f_0(e + \bar{y}_r, z) - \hat{\theta}^T f(e + \bar{y}_r, z) - \beta e^T P b [\|e + \bar{y}_r\|^2 + \|z\|^2 + (\alpha_1^{-1}(2r))^2 + 1] \right\} \triangleq v(e, z, r, \bar{y}_R, \hat{\theta}) \quad (16)$$

where  $\alpha_1^{-1}$  is the inverse function of  $\alpha_1$  and is again a function of class  $K_\infty$ ;  $\hat{\theta}$  is the estimation of  $\theta$ ;  $P$  is a matrix satisfying

$$P A_m + A_m^T P = -Q, \quad Q = Q^T > 0 \quad (17)$$

and  $\beta > 0$  is a design constant, which is the coefficient of the nonlinear damping term used to counter the effects of unmodelled dynamics.

Let us denote

$$\phi = 2e^T P b [f(e + \bar{y}_r, z) + gv] \triangleq \phi(e, z, r, \bar{y}_R, \hat{\theta}) \quad (18)$$

We use the adaptive law with smoothed projection [1] to obtain the parameter estimation  $\hat{\theta}$ , i.e.

$$\dot{\hat{\theta}} = P_{roj}(\hat{\theta}, \phi) \quad (19)$$

$$(P_{roj}(\hat{\theta}, \phi)) = \begin{cases} \gamma_{ii} \phi_i & \text{if } a_i \leq \hat{\theta}_i \leq b_i \text{ or} \\ & \text{if } \hat{\theta}_i > b_i \text{ and } \phi_i \leq 0 \text{ or} \\ & \text{if } \hat{\theta}_i < a_i \text{ and } \phi_i \geq 0 \\ \gamma_{ii} \left[ 1 + \frac{b_i - \hat{\theta}_i}{\delta} \right] \phi_i & \text{if } \hat{\theta}_i > b_i \text{ and } \phi_i > 0 \\ \gamma_{ii} \left[ 1 + \frac{\hat{\theta}_i - a_i}{\delta} \right] \phi_i & \text{if } \hat{\theta}_i < a_i \text{ and } \phi_i < 0 \end{cases} \quad (20)$$

where  $\gamma_{ii} > 0$ . Denote  $\Omega_\delta = (\theta) a_i - \delta \leq \theta_i \leq b_i + \delta$ ,  $1 \leq i \leq p$  where  $\delta > 0$  is a constant to be chosen such that  $\Omega_\delta \subset \Omega$ . The adaptive law (19) has the following properties [1]:

- $\hat{\theta}(t) \in \Omega_\delta, \forall t \geq 0$  if  $\hat{\theta}(0) \in \Omega$
- $\hat{\theta}^T \Gamma^{-1}(\hat{\theta} - \Gamma \phi) \leq 0$ , where  $\Gamma$  is a diagonal matrix whose diagonal element is  $\gamma_{ii}$
- $P_{roj}(\hat{\theta}, \phi)$  is locally Lipschitz in  $(\hat{\theta}, \phi)$

**Theorem 1:** Under assumptions 1–4, with the adaptive controller given by (14), (16) and (19), for any given  $x(0) \in Y, z(0) \in Z_0, \hat{\theta}(0) \in \Omega$ , all the variables of the closed-loop system are bounded in the presence of unmodelled dynamics and bounded disturbances. Furthermore, the mean-square tracking error can be made arbitrarily small if the design parameter  $\beta$  is chosen to be appropriately large. In addition, the tracking error converges to zero in the absence of unmodelled dynamics and disturbances.

The proof of theorem 1 is omitted here due to the limited space, but is available upon request.

To facilitate the discussion in the next Section and to prepare for the design of the adaptive output controller, we saturate  $r_m, v$  and  $\phi$  outside the domain of interest. This requires some *a priori* information about the system.

**Assumption 5:** For any  $e(0) \in E_0, \hat{\theta}(0) \in \Omega, z(0) \in Z_0, \omega(0) \in W_0, r(0) \in R_0^+, \bar{y}_R(0) \in Y_R$ , where  $E_0, \Omega, Z_0, W_0, Y_R$  are defined as before, and  $R_0^+$  is a compact subset in  $R^+$ , using the adaptive state feedback controller presented in this paper, we have  $e(t) \in E, \hat{\theta}(t) \in \Omega_\delta, z(t) \in Z, r(t) \in R_0, \bar{y}_R(t) \in Y_R \forall t \geq 0$ , where  $E \subset R^n, \Omega_\delta \subset R^p, Z \subset R^m, R_0 \subset R^+$  and  $Y_R \subset R^{n+1}$  are known compact sets.

Denote  $R_S = (e \in E) \times (z \in Z) \times (r \in R_0) \times \{\bar{y}_R \in Y_R\} \times \{\hat{\theta} \in \Omega_\delta\}$

Let

$$M^r \geq \max_{e \in E} |r_m(e, \bar{y}_r)| \quad (21)$$

$$M^v \geq \max_{(e, z, r, \bar{y}_R, \hat{\theta}) \in R_S} |v(e, z, r, \bar{y}_R, \hat{\theta})| \quad (22)$$

$$M_i^\phi \geq \max_{(e, z, r, \bar{y}_R, \hat{\theta}) \in R_S} |\phi_i(e, z, r, \bar{y}_R, \hat{\theta})|, \quad i = 1, \dots, p \quad (23)$$

We saturate  $r_m, v$  and  $\phi$  as follows:

$$r_m^s(e, \bar{y}_r) = M^r \cdot \text{sat}\left(\frac{r_m(e, \bar{y}_r)}{M^r}\right) \quad (24)$$

$$\phi_i^s(e, z, r, \bar{y}_R, \hat{\theta}) = M_i^\phi \cdot \text{sat}\left(\frac{\phi_i(e, z, r, \bar{y}_R, \hat{\theta})}{M_i^\phi}\right) \quad i = 1, \dots, p \quad (25)$$

$$v^s(e, z, r, \bar{y}_R, \hat{\theta}) = M^v \cdot \text{sat}\left(\frac{v(e, z, r, \bar{y}_R, \hat{\theta})}{M^v}\right) \quad (26)$$

where  $\text{sat}(\cdot)$  is the saturation function defined in [1].

Notice that in the case of state feedback,  $r_m^s = r_m, v^s = v, \phi^s = \phi$ , i.e. the saturation functions will not be effective and will not change the performance and the stability established in theorem 1.

#### 4 Robust adaptive control via output feedback

To implement the robust adaptive controller presented in Section 3 by output feedback, we need to design a state observer. Since the high-gain observers have the properties of rejecting disturbances, and allowing for uncertainties in modelling the systems [12–14], we adopt the following high-gain observer [1] to estimate  $e$ :

$$\begin{aligned} \dot{\hat{e}}_i &= \hat{e}_{i+1} + (\sigma_i/\varepsilon^i)(e_1 - \hat{e}_1), \quad 1 \leq i \leq n-1 \\ \dot{\hat{e}}_n &= (\sigma_n/\varepsilon^n)(e_1 - \hat{e}_1) \end{aligned} \quad (27)$$

where  $\varepsilon > 0$  is a small constant;  $\sigma_i > 0, i = 1, \dots, n$  are chosen to be the coefficients of a Hurwitz polynomial

$$\psi(\sigma_1, \dots, \sigma_n, s) = s^n + \sigma_1 s^{n-1} + \dots + \sigma_{n-1} s + \sigma_n \quad (28)$$

The main problem with a high-gain observer is that the states of the observer exhibit a peaking phenomenon in their transient behaviour. Such peaking states act as

destabilising inputs to the system [15]. To overcome the peaking, we use the following methods. First, to eliminate the peaking in the implementation of the observer, we define

$$\hat{e}_i = \frac{q_i}{\varepsilon^{i-1}}, \quad 1 \leq i \leq n \quad (29)$$

Then, (27) becomes

$$\begin{aligned} \varepsilon \dot{q}_i &= q_{i+1} + \sigma_i(e_1 - q_1) \quad 1 \leq i \leq n-1 \\ \varepsilon \dot{q}_n &= \sigma_n(e_1 - q_1) \end{aligned} \quad (30)$$

Secondly, to prevent the peaking from entering the control system, we saturate  $r_m, v$  and  $\phi$ . Thus, the robust adaptive controller via output feedback is given by

$$\dot{\hat{\theta}} = P_{roj}(\hat{\theta}, \phi^s(\hat{e}, z, r, \bar{y}_R, \hat{\theta})) \quad (31)$$

$$\dot{r} = -\bar{c}_0 r + r_m^s(\hat{e}, \bar{y}_r), \quad r(0) = r^0 > 0 \quad (32)$$

$$\dot{z} = \bar{A}z + \bar{b}v^s(\hat{e}, z, r, \bar{y}_R, \hat{\theta}) \quad (33)$$

$$u = z_1 \quad (34)$$

where  $\phi^s(\hat{e}, z, r, \bar{y}_R, \hat{\theta}) = [\phi_1^s, \dots, \phi_p^s]^T$ . We have the following results.

Under assumptions 1–5, with the adaptive controller presented by (31)–(34), for any  $e(0) \in E_0, \hat{\theta}(0) \in \Omega, z(0) \in Z_0, \omega(0) \in W_0, r(0) \in R_0^+$ , there exists  $\varepsilon_0 > 0$  such that for all  $0 < \varepsilon < \varepsilon_0$ , all the variables of the closed-loop system are bounded in the presence of unmodelled dynamics and bounded disturbances. Furthermore, the mean-square tracking error is of order  $O(\varepsilon)$  if the design parameter  $\beta$  is chosen to be appropriately large.

The proof of the above results, which is similar to that in [1], is omitted here due to the limited space, but is available upon request.

## 5 An example

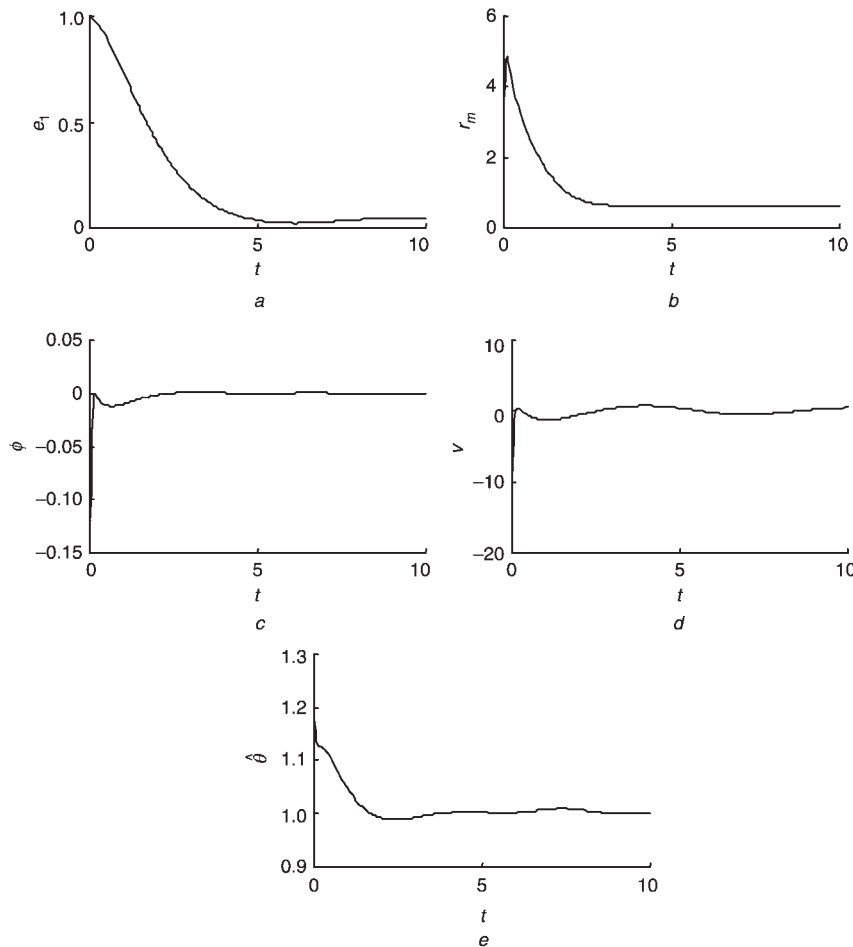
Consider a nonlinear system

$$y^{(3)} = (u + y - \ddot{y}) + 2\theta(y\dot{y} + \dot{y}^2 + y\ddot{y}) + \dot{u} + \Delta + d(t) \quad (35)$$

where  $d(t) = 0.5 \sin t$  is the disturbance and  $\Delta$  is the unmodelled dynamics given by

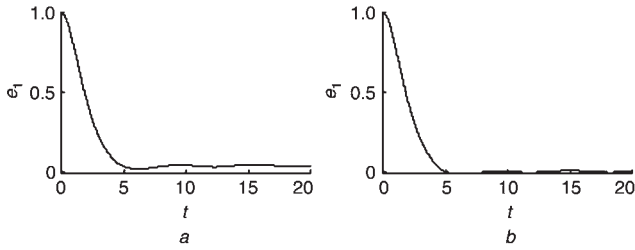
$$\dot{\omega} = -\omega + y^2 + \dot{y}^2 + 0.5, \quad \Delta = 2\omega \quad (36)$$

It can be checked directly that system (36) is exp-ISpS with



**Fig. 1** Simulation results by the adaptive state feedback controller

- a Tracking error  $e_1$
- b Signal  $r_m$
- c Signal  $\phi$
- d Control input  $v$  for the augmented system
- e Parameter estimation  $\hat{\theta}$



**Fig. 2** Simulation results by the adaptive output feedback controller

a Tracking error  $e_1$  when  $\beta = 20$   
b Tracking error  $e_1$  when  $\beta = 200$

$$\begin{aligned} V_\omega(\omega) &= \omega^2, \quad \alpha_1(\|\omega\|) = \|\omega\|^2, \\ \alpha_1^{-1}(s) &= \sqrt{s}, \quad \gamma(\|x\|) = 2.5\|x\|^4, \quad c_0 = 1.2, \\ d_0 &= 0.625 \end{aligned} \quad (37)$$

In the simulation, we assume that system (36) is unknown, but  $\alpha_1(\|\omega\|)$ ,  $\gamma(\|x\|)$ ,  $c_0$  and  $d_0$  are known. The objective is to design a robust adaptive controller such that  $y$  tracks  $y_r(t) = 0.1 \sin(t)$ .

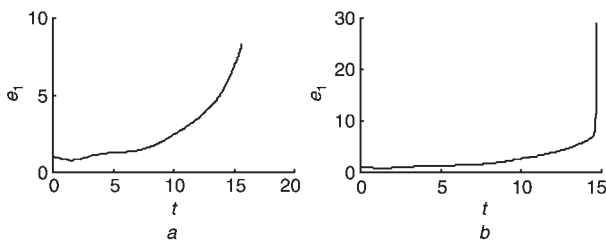
Take  $K = \begin{pmatrix} 2 & 4 & 3 \end{pmatrix}$ , and  $Q = I$ . Solving  $PA_m + A_m^T P = -I$ , we obtain  $P$ . In the simulation, the actual value of  $\theta$  is taken as  $\theta = 1$ . We assume  $\Omega = (\theta \mid 0 \leq \theta \leq 2)$ . The projection rule (20) is used to estimate  $\theta$  with  $\delta = 0.1$ ,  $\gamma = 10$ .

In the case of state feedback, with the following choice of the initial conditions and design parameters:

$$\begin{aligned} e(0) &= [1, 0, 0]^T, \quad z(0) = \omega(0) = 0, \quad \hat{\theta}(0) = 1.2, \\ r(0) &= 1, \quad \bar{c}_0 = 0.6, \quad \beta = 20 \end{aligned} \quad (38)$$

the simulation results are given in Fig. 1.

By analysis as in [1] or by simulations, we can estimate the upper bounds on  $r_m$ ,  $v$  and  $\phi$  under the state feedback adaptive controller. For simplicity, we use the simulation results given above. Based on Fig. 1b–d, we take  $M^r = 7$ ,  $M^\phi = 0.15$ ,  $M^v = 30$ . Using (24)–(26), we obtain  $r_m^s$ ,  $\phi^s$  and  $v^s$  for the robust adaptive output feedback controller. With  $\varepsilon = 0.01$  and the rest of design parameters and all the initial conditions being the same as in (38), the results are shown in Fig. 2a. Next, we increased the value of  $\beta$  to  $\beta = 200$  with the rest of design parameters and initial conditions being the same as in (38), and obtained the simulation results of the adaptive output feedback controller shown in Fig. 2b. It can be seen that the tracking error in Fig. 2b is very small. This illustrates that the mean-square tracking errors can be



**Fig. 3** Simulation results by the adaptive controller presented in [1]

a Tracking error  $e_1$  via state feedback  
b Tracking error  $e_1$  via output feedback

made arbitrarily small by choosing  $\beta$  to be appropriately large.

Finally, to compare the robust adaptive control scheme presented in this paper with that in [1], we applied the adaptive control scheme presented in [1] to system (35) with the following initial conditions  $e(0) = [1, 0, 0]^T$ ,  $z(0) = \omega(0) = 0$ ,  $\hat{\theta}(0) = 1.2$ . The simulation results of the state feedback and the output feedback are shown in Fig. 3a and b, respectively, from which we can see that the tracking errors are unbounded and the adaptive control scheme presented in [1] is not robust to non-small unmodelled dynamics.

## 6 Conclusions

Using a dynamic signal and introducing a nonlinear damping term, this paper has presented a robust adaptive output feedback controller for a class of nonlinear systems represented by input–output models. Under certain assumptions, the proposed adaptive controller guarantees that all the variables of the closed-loop system are bounded in the presence of unmodelled dynamics and bounded time-varying disturbances and the mean-square tracking error can be made arbitrarily small by choosing the design parameters appropriately. Simulation results show that the adaptive controller given in this paper is very effective.

It should be pointed out that the model (1) has a restriction of linear dependence on the unknown parameters and assumptions 2 and 3 also place some restrictions to the applications of the proposed scheme. Although the authors have removed the restriction of linear dependence along another research line [16], which is free of parameter estimation, how to relax these restrictions along the research line of parameter estimation is still a subject for further research.

## 7 Acknowledgment

The authors acknowledge the support from the National Key Basic Research Special Fund of China under grant G 1998020311 and the support from the National Natural Science Foundation of China under grant No. 60374012.

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