Robust Adaptive Control of Nonlinear Systems Represented by Input–Output Models

Yusheng Liu and Xing-Yuan Li

***Abstract*—A robust adaptive control scheme is proposed for a class of nonlinear systems represented by input–output models with unmodeled dynamics. The scheme does not require the unknown parameters to satisfy the linear dependence condition and parameter estimation is not needed. With the proposed control scheme, all the variables in the closed-loop system are bounded in the presence of unmodeled dynamics and bounded disturbances. Moreover, the mean-square tracking error can be made arbitrarily small by choosing some design parameters appropriately.**

***Index Terms*—Adaptive control, nonlinear systems, output feedback, robustness, unmodeled dynamics.**

I. INTRODUCTION

Various adaptive control schemes have been proposed for nonlinear systems. In [1], an adaptive output feedback controller is presented for a class of nonlinear systems represented by input–output models The adaptive controller in [1] has several advantages: it applies to a wide class of nonlinear systems; the models of the systems may depend nonlinearly on control input u; and filtering or error augmentation is not required. However, the adaptive controller is not robust to unmodeled dynamics.

By adding a robustifying control component, the results of [1] are extended to the case with bounded disturbance in [2], but an upper bound on the disturbance must be known. With significant progress made in the robust adaptive control of nonlinear systems with parameter uncertainty and uncertain nonlinearities [3]–[5], robust adaptive control for nonlinear systems with unmodeled dynamics has received great attention [6]–[12]. However, few results are available on the robust adaptive control of the nonlinear systems represented by input–output models with unmodeled dynamics. Using the approach of [8], the authors of [12] extended their previous work to the case with unmodeled dynamics; however, as in [1], [2], the parameters of the system must satisfy the linear dependence condition. In [9], an adaptive H^∞ tracking control scheme is presented for nonlinear multiple-input–multiple-output systems with unmodeled perturbations, but the input–output models depend linearly on control inputs u_i and unmodeled dynamics from other systems is not considered.

On the other hand, in the robust control of nonlinear systems with uncertainties, a bounding function is often employed to ensure the stability of the systems. To determine the bounding function, some knowledge about the system is used to estimate the size of the uncertainty. Due to the nature of uncertainties, overestimation is inevitable, which causes the robust control conservative in the sense that its gain is unnecessarily large. One of the approaches to maintain robustness while reducing conservatism is to blend adaptive control scheme into robust control design by constructively using Lyapunov theory, as suggested in [17], [18], and the references therein. There are at least two ways to do this. In [19], we first employ an adaptive law with smoothed projection as in [1], [2], and [12] to estimate the unknown parameters, then design a robust controller to guarantee the stability of the totally closed-loop system. However, such kinds of schemes will take longer time in computation and sometimes are not suitable for real-time control. Besides, to reduce conservatism, we have to estimate the parameters accurately, which needs some additional requirements such as the linear dependence condition, persistent excitation etc.

In this work, we use a high-gain robust nonlinear controller in which the high-gain parameter is adaptively chosen on line. Such learning process on line reduces conservatism and prevents the gain from being unnecessarily large. Based on this idea, we present a robust adaptive output feedback controller for a class of nonlinear systems represented by input–output models containing unmodeled dynamics and bounded time-varying disturbances. The scheme does not require the unknown parameters to satisfy the linear dependence condition and parameter estimation is not needed. With the proposed control scheme, all the variables in the closed-loop system are bounded in the presence of unmodeled dynamics and bounded disturbances. Moreover, the mean-square tracking error can be made arbitrarily small by choosing some design parameters appropriately.

II. PROBLEM STATEMENT

We consider a single-input–single-output nonlinear system described by

$$y^{(n)} = f\left(y,\, \dot{y},\, \ldots,\, y^{(n-1)},\, u,\, \dot{u},\, \ldots,\, u^{(m-1)}\right) + \theta_u u^{(m)} + \Delta\left(y,\, \dot{y}, \ldots, y^{(n-1)},\, u,\, \dot{u}, \ldots, u^{(m-1)},\, \omega\right) + d(t) \tag{1}$$

where y is the output; u is the control; $y^{(i)}$ is the ith derivative of y; $d(t)$ is the unknown bounded disturbance; $\Delta(\cdot)$ represents the uncertain nonlinearity and the unmodeled dynamics described later; and θ_u is an unknown constant parameter, but the sign of θ_u is known. Without loss of generality, we assume that $\theta_u > 0$. It is assumed in (1) that f is an unknown smooth nonlinear function satisfying

$$\left| f\left(y,\, \dot{y},\, \ldots,\, y^{(n-1)},\, u,\, \dot{u},\, \ldots,\, u^{(m-1)}\right)\right| \le \theta \overline{f}\left(y,\, \dot{y},\, \ldots,\, y^{(n-1)},\, u,\, \dot{u},\, \ldots,\, u^{(m-1)}\right)$$

where $\overline{f}(y,\, \dot{y},\, \ldots,\, y^{(n-1)},\, u,\, \dot{u},\, \ldots,\, u^{(m-1)})$ is a known smooth nonlinear function, $\theta > 0$ is an unknown constant.

Remark: The systems considered in [1], [2], [12], and [19] have a restriction of linear dependence on the unknown parameters. It can be seen from (1) that the linear dependence condition has been removed here.

Let

$$x_1 = y,\, x_2 = y^{(1)},\, \ldots,\, x_n = y^{(n-1)}$$
$$z_1 = u,\, z_2 = u^{(1)},\, \ldots,\, z_m = u^{(m-1)}.$$

We have

$$\begin{aligned} \dot{x}_i &= x_{i+1}, \qquad 1 \le i \le n-1 \\ \dot{x}_n &= f(x,\, z) + \theta_u v + \Delta(x,\, z,\, \omega) + d(t) \\ \dot{z}_i &= z_{i+1}, \qquad 1 \le i \le m-1 \\ \dot{z}_m &= v \end{aligned} \tag{2}$$

where $v = u^{(m)}$ is the control input for the augmented system (2), and $x = [x_1,\, \ldots,\, x_n]^T$, $z = [z_1,\, \ldots,\, z_m]^T$. The $\omega \in R^l$ in $\Delta(x,\, z,\, \omega)$ is the unmodeled dynamics described by

$$\dot{\omega} = q(\omega,\, x,\, z). \tag{3}$$

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We assume that Δ and q are unknown nonlinear functions which are continuous and Lipschitz and satisfy

$$|\Delta(x,\, z,\, \omega)| \le c_1\|x\| + c_2\|z\| + c_3\|\omega\| \tag{4}$$
$$\Delta(0,\, z,\, 0) = 0 \tag{5}$$

where $c_1,\, c_2,\, c_3 \ge 0$ are unknown constants. In (4) and throughout this note, we use the Euclidean norm. Let

$$\zeta_i = z_i - \frac{x_{n-m+i}}{\theta_u}, \qquad 1 \le i \le m. \tag{6}$$

From (2), we have

$$\begin{aligned}\dot{\zeta}_i &= \zeta_{i+1} \qquad 1 \le i \le m-1\\ \dot{\zeta}_m &= -\left.\frac{f(x,z) + \Delta(x,z,\omega) + d(t)}{\theta_u}\right|_{z_i=\zeta_i+(x_{n-m+i}/\theta_u)}.\end{aligned} \tag{7}$$

Let

$$\zeta = [\zeta_1,\, \ldots,\, \zeta_m]^T \qquad \bar{b} = [0,\, \ldots,\, 0,\, 1]^T$$
$$h(\zeta,\, x) = \left[\zeta_2,\, \ldots,\, \zeta_{m-1},\, -\frac{f(x,\, z)}{\theta_u}\right]^T.$$

We have

$$\dot{\zeta} = h(\zeta,\, x) + \bar{b}\left(\frac{-\Delta(x,z,\omega) - d(t)}{\theta_u}\right). \tag{8}$$

Thus, the nominal system of (2) consists of the first n equations in (2) and of (8) with $\Delta(x,\, z,\, \omega) = 0$, $d(t) = 0$.

We further assume that the reference signal $y_r(t)$ is bounded with bounded derivatives up to the nth order and $y_r^{(n)}(t)$ is piecewise continuous. Denote

$$\bar{y}_r = \left[y_r,\, y_r^{(1)},\, \ldots,\, y_r^{(n-1)}\right]^T$$
$$\bar{y}_R = \left[y_r,\, y_r^{(1)},\, \ldots,\, y_r^{(n)}\right]^T.$$

Let $Y \subset R^n$, $Y_R \subset R^{n+1}$, $Z_0 \subset R^m$, $W_0 \subset R^l$ be any given compact sets. Then, the objective of this note is to design a robust adaptive output feedback controller for (2) and (3) such that for any $x(0) \in Y$, $z(0) \in Z_0$, $\omega(0) \in W_0$ and $\bar{y}_R \in Y_R$, the output $y(t)$ of the system tracks the reference signal $y_r(t)$ and all the variables of the closed-loop system are bounded in the presence of unmodeled dynamics and bounded disturbances. We need the following assumptions.

Assumption 1: In the nominal system, the subsystem $\dot{\zeta} = h(\zeta,\, x)$ has a unique steady-state solution $\bar{\zeta}$ [2]. Without loss of generality, we assume $\bar{\zeta} = 0$. Moreover, the subsystem has a function $w(t,\, \zeta)$ satisfying

$$\begin{aligned}\pi_1\|\zeta\|^2 \le w(t,\, \zeta) &\le \pi_2\|\zeta\|^2\\ \frac{\partial w}{\partial t} + \frac{\partial w}{\partial \zeta}\, h(\zeta,\, x) &\le -\pi_3\|\zeta\|^2 + \pi_4\|\zeta\|\|x\|\\ \left\|\frac{\partial w}{\partial \zeta}\right\| &\le \pi_5\|\zeta\|\end{aligned} \tag{9}$$

where $\pi_i > 0$, $i = 1,\, \ldots,\, 5$, are constants and $\pi_3 > \pi_5 c_2$.

Assumption 2: The unmodeled dynamics, i.e., the system of (3), is exponentially input-to-state practically stable (exp-ISpS) [8]; that is, (3) has an exp-ISpS Lyapunov function $V_\omega(\omega)$ satisfying

$$\alpha_1(\|\omega\|) \le V_\omega(\omega) \le \alpha_2(\|\omega\|) \tag{10}$$
$$\frac{\partial V_\omega(\omega)}{\partial \omega}\, q(\omega,\, x,\, z) \le -c_0 V_\omega(\omega) + \gamma(\|x\|) + d_0 \tag{11}$$

where α_1, α_2 are functions of class K_∞ and $c_0 > 0$, $d_0 \ge 0$ are constants. Without loss of generality, we assume $\gamma(\cdot)$ has the form $\gamma(s) = s^2\gamma_0(s^2)$, where γ_0 is a nonnegative smooth function. Otherwise, as indicated in [8], it suffices to replace γ in (11) by $\|x\|^2\gamma_0(\|x\|^2) + \bar{\varepsilon}_0$ with $\bar{\varepsilon}_0 > 0$ being a sufficiently small real number.

III. Robust Adaptive Control Scheme

In this section, we first assume that the state $(x,\, z)$ of system (2) is available for feedback. Let $e_1 = x_1 - y_r$, $e_2 = x_2 - \dot{y}_r,\, \ldots,\, e_n = x_n - y_r^{(n-1)}$, $e = [e_1,\, e_2,\, \ldots,\, e_n]^{\mathrm{T}}$. Using (2) and (3), we get

$$\begin{aligned}\dot{e} &= Ae + b\left[f(e+\bar{y}_r,\, z) + \theta_u v - y_r^{(n)} + \Delta(e+\bar{y}_r,\, z,\, \omega) + d(t)\right]\\ \dot{z} &= \bar{A}z + \bar{b}v\\ \dot{\omega} &= q(\omega,\, e+\bar{y}_r,\, z)\end{aligned} \tag{12}$$

where

$$A = \begin{bmatrix}0 & 1 & \cdots & 0\\ \vdots & \vdots & & \vdots\\ 0 & 0 & \cdots & 1\\ 0 & 0 & \cdots & 0\end{bmatrix} \qquad b = \begin{bmatrix}0\\ \vdots\\ 0\\ 1\end{bmatrix}$$

and $\bar{A}$, $\bar{b}$ have the same forms as A, b but with different sizes. Let $A_m = A - bK$, where K is chosen so that A_m is Hurwitz. Then, we have

$$\begin{aligned}\dot{e} = A_m e + b\Big[&Ke + f(e+\bar{y}_r,\, z) - y_r^{(n)} + \theta_u v\\ &+ \Delta(e+\bar{y}_r,\, z,\, \omega) + d(t)\Big].\end{aligned} \tag{13}$$

To characterize the effects of the ummodeled dynamics to the control system, we use a dynamic signal described by

$$\dot{r} = -\bar{c}_0 r + r_m(e,\, \bar{y}_r), \qquad r(0) = r^0 > 0 \tag{14}$$

where $\bar{c}_0 \in (0,\, c_0)$, $r_m(e,\, \bar{y}_r) = \|e+\bar{y}_r\|^2\gamma_0(\|e+\bar{y}_r\|^2) + d_0$. It has been shown in [8] that the signal has the following property:

$$V_\omega(\omega(t)) \le r(t) + D(t) \tag{15}$$

for all $t \ge 0$ where the solutions are defined, with $D(t)$ defined for $t \ge 0$ and there is a finite T^0 such that $D(t) = 0$ for all $t \ge T^0 \ge 0$.

We propose the following robust adaptive controller:

$$\begin{aligned}v = &-\beta e^T Pb\left\{\left[\bar{f}(e+\bar{y}_r,\, z)\right]^2 + \|e+\bar{y}_r\|^2 + \|z\|^2\right.\\ &\left.+ \left[\alpha_1^{-1}(2r)\right]^2 + (Ke)^2 + 1\right\}\\ \triangleq\, &v(e,\, z,\, r,\, \bar{y}_r,\, \beta)\end{aligned} \tag{16}$$

where α_1^{-1} is the inverse function of α_1 and is again a function of class K_∞; P is a matrix satisfying

$$PA_m + A_m^T P = -Q, \qquad Q = Q^T > 0. \tag{17}$$

In (16), β is the adaptive parameter of the controller. We present the following adaptive law:

$$\dot{\beta} = \beta_m(e,\, z,\, r,\, \bar{y}_r) - \Gamma\sigma\beta \tag{18}$$

where

$$\begin{aligned}\beta_m(e,\, z,\, r,\, \bar{y}_r) = \Gamma(e^T Pb)^2\{&[\bar{f}(e+\bar{y}_r,\, z)]^2 + \|e+\bar{y}_r\|^2\\ &+ \|z\|^2 + [\alpha_1^{-1}(2r)]^2 + (Ke)^2 + 1\}\end{aligned}$$

and $\Gamma > 0$, $\sigma > 0$ are design constants.

Theorem 1: Under Assumptions 1 and 2, with the proposed adaptive state feedback controller, for any given $x(0) \in Y$, $z(0) \in Z_0$, $\omega(0) \in W_0$ and $\bar{y}_R \in Y_R$, all the variables of the closed-loop system are bounded in the presence of unmodeled dynamics and bounded disturbances. Furthermore, the mean-square tracking error can be made arbitrarily small by choosing the design parameters Γ, σ and Q appropriately.

Proof: Consider the Lyapunov function candidate

$$V = e^T Pe + \theta_u\Gamma^{-1}(\beta - \beta^*)^2 \tag{19}$$

where $\beta^* > 0$ is a constant, which is the desired value of β, i.e., when $\beta = \beta^*$, the controlled system has a desired performance. Taking the time derivative of V along the solutions of (13) and (18) yields

$$\begin{aligned}\dot{V} \leq & -e^T Qe + 2\left|e^{\mathrm{T}} Pb\right| \cdot |Ke| + 2\left|e^{\mathrm{T}} Pb\right| \cdot \left[\theta \overline{f}\,(e + \overline{y}_r,\, z)\right.\\ & \left. + c_1 \|e + \overline{y}_r\| + c_2\|z\| + c_3\|\omega\| + \left|y_r^{(n)}(t)\right| + |d(t)|\right]\\ & - 2\theta_u \beta^* (e^T Pb)^2 \left\{\left[\overline{f}\,(e + \overline{y}_r,\, z)\right]^2 + \|e + \overline{y}_r\|^2\right.\\ & \left. + \|z\|^2 + \left[\alpha_1^{-1}(2r)\right]^2 + (Ke)^2 + 1\right\} - \theta_u \sigma \beta^2 + \theta_u \sigma \beta^{*2}\\ & - \theta_u \sigma (\beta - \beta^*)^2. \end{aligned} \tag{20}$$

From (10) and (15), we get

$$\begin{aligned} 2c_3\left|e^T Pb\right| \cdot \|\omega\| & \leq 2c_3\left|e^T Pb\right| \alpha_1^{-1}(r + D(t))\\ & \leq 2c_3\left|e^T Pb\right| \alpha_1^{-1}(2r) + 2c_3\left|e^T Pb\right| \alpha_1^{-1}(2D(t)). \end{aligned} \tag{21}$$

Notice that $D(t) = 0,\ \forall t \geq T^0$, thus, $2c_3\alpha_1^{-1}(2D(t)) = 0,\ \forall t \geq T^0$. Let

$$c_4 = \sup\left\{\left|y_r^{(n)}(t)\right| + |d(t)| + c_3\alpha_1^{-1}(2D(t))\right\}. \tag{22}$$

Using (21) and (22) gives

$$\begin{aligned}\dot{V} \leq & -e^T Qe - 2\beta^*\theta_u \left(\left|e^T Pb\right| \cdot |Ke| - \frac{1}{2\beta^*\theta_u}\right)^2 + \frac{1}{2\beta^*\theta_u}\\ & - 2\beta^*\theta_u \left(\left|e^T Pb\right| \cdot \overline{f}(e + \overline{y}_r,\, z) - \frac{\theta}{2\beta^*\theta_u}\right)^2\\ & + \frac{\theta^2}{2\beta^*\theta_u} - 2\beta^*\theta_u \left(\left|e^T Pb\right| \cdot \|e + \overline{y}_r\| - \frac{c_1}{2\beta^*\theta_u}\right)^2\\ & + \frac{c_1^2}{2\beta^*\theta_u} - 2\beta^*\theta_u \left(\left|e^T Pb\right| \cdot \|z\| - \frac{c_2}{2\beta^*\theta_u}\right)^2\\ & + \frac{c_2^2}{2\beta^*\theta_u} - 2\beta^*\theta_u \left(\left|e^T Pb\right| \cdot \alpha_1^{-1}(2r) - \frac{c_3}{2\beta^*\theta_u}\right)^2\\ & + \frac{c_3^2}{2\beta^*\theta_u} - 2\beta^*\theta_u \left(\left|e^T Pb\right| - \frac{c_4}{2\beta^*\theta_u}\right)^2\\ & + \frac{c_4^2}{2\beta^*\theta_u} - \sigma\theta_u\beta^2 - \sigma\theta_u(\beta - \beta^*)^2 + \sigma\theta_u\beta^{*2}. \end{aligned} \tag{23}$$

Thus

$$\dot{V} \leq -e^T Qe - \sigma\theta_u(\beta - \beta^*)^2 + M_1 \tag{24}$$

where $M_1 = (1/2\beta^*\theta_u)(c_1^2 + c_2^2 + c_3^2 + c_4^2 + \theta^2 + 1) + \sigma\theta_u\beta^{*2}$. We have

$$\dot{V} \leq -\mu V + M_1 \tag{25}$$

where

$$\mu = \min\left\{\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)},\, \theta_u\sigma_\Gamma\right\}. \tag{26}$$

Therefore, $V(e,\,\beta)$ decreases monotonically until $(e,\,\beta)$ reaches the compact set $C_s = \left\{(e,\,\beta) \in R^n \times R\colon V(e,\,\beta) \leq \mu^{-1}M_1\right\}$. This means that $(e,\,\beta)$ are bounded. Since e and $\overline{y}_r$ are bounded, x is bounded. Thus, from (14), $r(t)$ is bounded. Then, the boundedness of ω is followed by using (15).

Differentiating the function $w(t,\,\zeta)$ in Assumption 1 along the solution of (8) gives

$$\begin{aligned}\dot{w}(t,\,\zeta) \leq & -\pi_3\|\zeta\|^2 + \pi_4\|\zeta\|\,\|x\| + \pi_5\|\zeta\|\\ & \cdot \left(c_1\|x\| + c_2\|\zeta\| + c_2\|x\| \cdot \left|\frac{1}{\theta_u}\right| + c_3\|\omega\| + |d(t)|\right). \end{aligned} \tag{27}$$

Since x, ω, $d(t)$ are bounded, and $\pi_3 > \pi_5 c_2$, it can be seen from (27) that ζ is bounded; therefore, z is bounded. We conclude that all the variables of the closed-loop system are bounded.

Furthermore, it can be seen from (26) that choosing Q and the design constants σ, Γ appropriately will reduce the residual error bound $\mu^{-1}M_1$, and make the tracking error arbitrarily small. □

To implement the robust adaptive controller presented above by output feedback, we need to design a state observer. Since the high-gain observers have the properties of rejecting disturbances and allowing for uncertainties in modeling the systems [13]–[16], we adopt the following high-gain observer [1] to estimate e:

$$\begin{aligned} \dot{\hat{e}}_i &= \hat{e}_{i+1} + (\sigma_i/\varepsilon^i)(e_1 - \hat{e}_1), \qquad 1 \leq i \leq n-1\\ \dot{\hat{e}}_n &= (\sigma_n/\varepsilon^n)(e_1 - \hat{e}_1) \end{aligned} \tag{28}$$

where $\varepsilon > 0$ is a small constant; $\sigma_i > 0,\ i = 1, \ldots, n$ are chosen so that $A_n = A - K_\sigma C$ is a Hurwitz matrix, where $K_\sigma = [\sigma_1,\, \sigma_2,\, \ldots,\, \sigma_n]^T$, $C = [1,\, 0,\, \ldots,\, 0]$.

To eliminate peaking in the implementation of the observer [1], we define

$$\hat{e}_i = \frac{q_i}{\varepsilon^{i-1}}, \qquad 1 \leq i \leq n. \tag{29}$$

Then, (28) becomes

$$\begin{aligned} \varepsilon\dot{q}_i &= q_{i+1} + \sigma_i(e_1 - q_1), \qquad 1 \leq i \leq n-1\\ \varepsilon\dot{q}_n &= \sigma_n(e_1 - q_1). \end{aligned} \tag{30}$$

To prevent the peaking from entering the control system, we saturate r_m, β_m and v outside their domains of interests. As in [1], we need some *a priori* information about the system.

Assumption 3: $\overline{\theta} = [\theta,\, \theta_u]^{\mathrm{T}} \in \Omega$, $d(t) \in \Phi$, where Ω and Φ are known compact sets.

Assumption 4: For any

$$e(0) \in E_0,\ z(0) \in Z_0,\ \omega(0) \in W_0,\ r(0) \in R_0^+,\ \overline{y}_R(0) \in Y_R,\ \beta(0) \in R_1^0$$

where E_0, Z_0, W_0, Y_R are defined as before, and $R_0^+ \subset R^+$, $R_1^0 \subset R$ are compact sets, using the proposed adaptive state feedback controller, we have $e(t) \in E$, $z(t) \in Z$, $\omega(t) \in W$, $r(t) \in R_0$, $\overline{y}_R(t) \in Y_R$ $\beta(t) \in R_1\ \forall t \geq 0$, where $E \subset R^n$, $Z \subset R^m$, $W \subset R^l$, $R_0 \subset R^+$, $Y_R \subset R^{n+1}$ and $R_1 \subset R$ are known compact sets.

Denote

$$R_S = \{e \in E\} \times \{z \in Z\} \times \{\omega \in W\} \times \{r \in R_0\} \times \{\overline{y}_R \in Y_R\} \times \{\beta \in R_1\}.$$

Choose M_r, M_β and M_v be such constants that are larger than or equal to the upper bounds of $r_m(e,\,\overline{y}_r)$, $\beta_m(e,\,z,\,r,\,\overline{y}_r)$ and $v(e,\,z,\,r,\,\overline{y}_r,\,\beta)$ over R_S, respectively. Denote

$$r_m^s(\hat{e},\,\overline{y}_r) = M_r \cdot sat\left(\frac{r_m(\hat{e},\overline{y}_r)}{M_r}\right) \tag{31}$$

$$\beta_m^s(\hat{e},\,z,\,r,\,\overline{y}_r) = M_\beta \cdot sat\left(\frac{\beta_m(\hat{e},z,r,\overline{y}_r)}{M_\beta}\right) \tag{32}$$

$$v^s(\hat{e},\,z,\,r,\,\overline{y}_r,\,\beta) = M_v \cdot sat\left(\frac{v(\hat{e},z,r,\overline{y}_r,\beta)}{M_v}\right) \tag{33}$$

where $sat(\cdot)$ represents the saturation function defined in [1]. Thus, the robust adaptive output controller can be obtained by replacing $r_m(e,\,\overline{y}_r)$, $\beta_m(e,\,z,\,r,\,\overline{y}_r)$ and $v(e,\,z,\,r,\,\overline{y}_r,\,\beta)$ in (12), (14), (16), and (18) with $r_m^s(\hat{e},\,\overline{y}_r)$, $\beta_m^s(\hat{e},\,z,\,r,\,\overline{y}_r)$ and $v^s(\hat{e},\,z,\,r,\,\overline{y}_r,\,\beta)$, respectively. We have the following results.

Under Assumptions 1–4, with the proposed adaptive output controller, for any $e(0) \in E_0$, $z(0) \in Z_0$, $\omega(0) \in W_0$, $r(0) \in R_0^+$, $\beta(0) \in R_1^0$, there exists $\varepsilon_0 > 0$ such that for any $\varepsilon \in (0,\,\varepsilon_0)$, all the variables of the closed-loop system are bounded in the presence of unmodeled dynamics and bounded disturbances. Furthermore, the

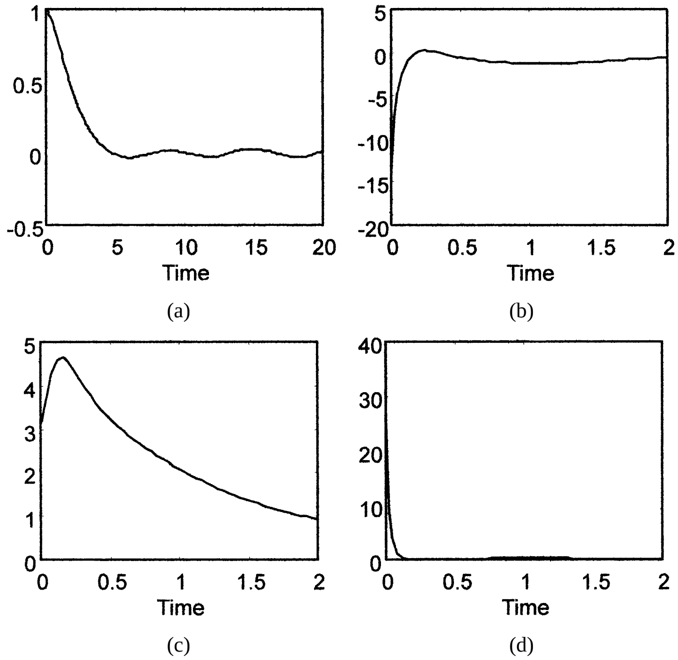

Fig. 1. Simulation results of the adaptive state feedback controller. (a) Tracking error. (b) Control input v for the augmented system. (c) Signal r_m. (d) Signal β_m.

mean-square tracking error is of order $O(\varepsilon)$ if the design parameters Γ, σ and Q are chosen appropriately.

The proof of the previous results, which is similar to that in [1], is omitted here due to the limited space, but is available upon request.

IV. Illustration Example

Consider a nonlinear system

$$y^{(3)} = \theta_0(u + y - \ddot{y}) + 2\theta_1(y\dot{y} + \dot{y}^2 + y\ddot{y}) + \theta_u \dot{u} + \Delta + d(t) \tag{34}$$

which is the same as [1, Ex. 3] except that we add Δ and $d(t)$ in the system. In (34), $d(t) = 0.5\sin t$ is the disturbance and Δ is the unmodeled dynamics given by

$$\dot{\omega} = -\omega + y^2 + \dot{y}^2 + 0.5 \quad \Delta = 2\omega. \tag{35}$$

It can be checked directly that (35) is exp-ISpS with

$$V_\omega(\omega) = \omega^2,\ \alpha_1(|\omega|) = |\omega|^2,\ \alpha_1^{-1}(s) = \sqrt{s}$$
$$\gamma(\|x\|) = 2.5\|x\|^4,\ c_0 = 1.2,\ d_0 = 0.625. \tag{36}$$

We assume that (35) is unknown, but $\alpha_1(\cdot),\ \gamma(\cdot),\ c_0$ and d_0 are known.

We further assume that in (34), $f(y, \dot{y}, \ddot{y}, u) = \theta_0(u + y - \ddot{y}) + 2\theta_1(y\dot{y} + \dot{y}^2 + y\ddot{y})$ is unknown but satisfying $|f(y, \dot{y}, \ddot{y}, u)| \leq \theta\overline{f}(y, \dot{y}, \ddot{y}, u)$ with $\overline{f}(y, \dot{y}, \ddot{y}, u) = [(u + y - \ddot{y})^2 + (y\dot{y} + \dot{y}^2 + y\ddot{y})^2]^{1/2}$ known. It is also assumed that θ and θ_u are unknown, but the sign of θ_u is known. The objective is to design a robust adaptive controller such that y tracks $y_r(t) = 0.1\sin(t)$.

Take $K = [\,2 \quad 4 \quad 3\,]$ and $Q = I$. Solving $PA_m + A_m^T P = -I$, we obtain P. For the purpose of simulation, we take $\theta_0 = \theta_1 = \theta_u = 1$.

With the following choice of the initial conditions and design parameters:

$$e(0) = [1,\ 0,\ 0]^T,\ z(0) = \omega(0) = 0,\ r(0) = 1,\ \beta(0) = 10,$$
$$\overline{c}_0 = 0.6,\ \Gamma = 100,\ \sigma = 0.0001 \tag{37}$$

the simulation results under the adaptive state feedback controller are given in Fig. 1.

By analysis as in [1] or by simulations, we can estimate the upper bounds on $v,\ r_m,\ \beta_m$ under the state feedback adaptive control. For simplicity, here we use the aforementioned simulation results. Based on Fig. 1(b)–(d), we take $M_v = 20,\ M_r = 5,\ M_\beta = 40$. Using (31)–(33), we get $v^s,\ r_m^s$ and β_m^s for the robust adaptive output feedback controller. Choose $\varepsilon = 0.01$ and the rest of design parameters and all the initial conditions to be the same as in (37), the simulation results under the adaptive output feedback controller are shown in Fig. 2.

Choosing $\Gamma = 10^5$, $\sigma = 10^{-7}$ and the rest of initial conditions and design parameters to be the same as in (37), the simulation results are shown in Fig. 3, which demonstrates that the mean-square tracking error can be made arbitrarily small under the state feedback adaptive

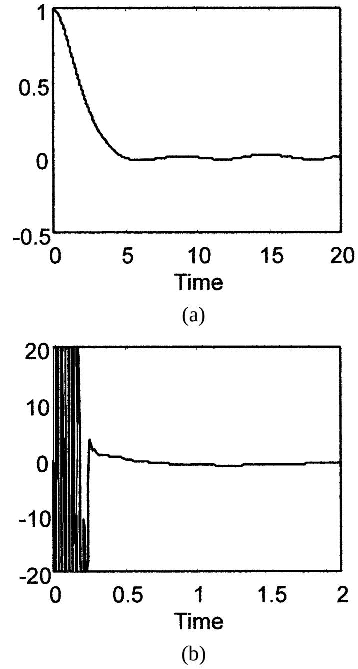

Fig. 2. Simulation results by the adaptive output feedback controller. (a) Tracking error e_1. (b) Control input v^s for the augmented system.

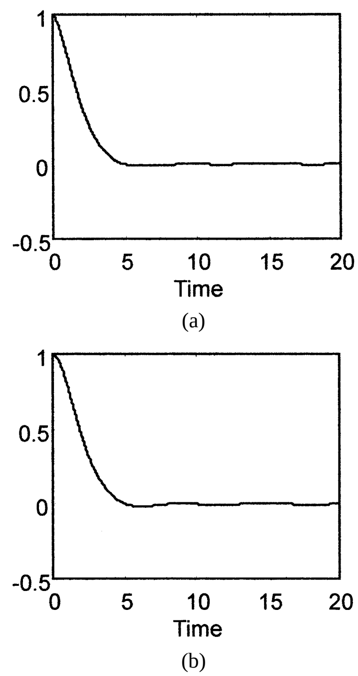

Fig. 3. Simulation results by choosing the design parameters appropriately ($\Gamma = 10^5$, $\sigma = 10^{-7}$). (a) Tracking error e_1 via state feedback. (b) Tracking error e_1 via output feedback.

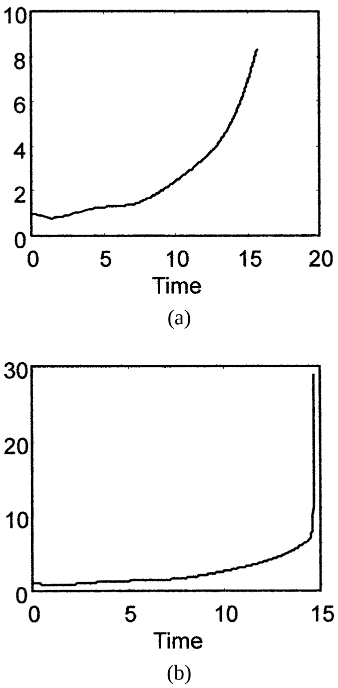

Fig. 4. Simulation results by the adaptive controller presented in [1]. (a) Tracking error e_1 via state feedback. (b) Tracking error e_1 via output feedback.

control and the mean-square tracking error is of order $O(\varepsilon)$ under the output feedback adaptive control by choosing the design parameters Γ, σ appropriately.

Finally, to compare the adaptive scheme presented in this note with that in [1], we applied the adaptive scheme of [1] to (34). As in [1], θ_0, θ_u are assumed to be known, but θ_1 is estimated by the projection law in [1] with $\delta = 0.1, \gamma = 10, \Omega = \{\theta_1 | 0 \leq \theta_1 \leq 2\}$ (using the same notations as in [1]). The initial conditions are $e(0) = [1, 0, 0]^T$, $z(0) = \omega(0) = 0$, $\hat{\theta}_1(0) = 1.2$. The simulation results of the state feedback and the output feedback are shown in Fig. 4(a) and (b), respectively, from which we can see that the tracking errors are unbounded and the adaptive control scheme presented in [1] is not robust to unmodeled dynamics.

V. Conclusion

This work presents a robust adaptive control scheme for a class of nonlinear systems represented by input–output models. It has been shown that under certain assumptions, the proposed adaptive controller guarantees that all the variables of the closed-loop system are bounded in the presence of unmodeled dynamics and bounded time-varying disturbances and the mean-square tracking error can be made arbitrarily small by choosing the design parameters appropriately. The scheme does not require the unknown parameters to satisfy the linear dependence condition and no parameter estimation is needed. Simulation results show that the adaptive controller of this note is very effective.

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