Instructions:

1. **Clear your desk** of all but an equation sheet and writing utensils.
2. Put your name on **all separate sheets** turned in.
3. Draw a circle, underline, box, etc., around your **final answer** in all cases. If more than one answer is provided, I will grade the least correct.
4. **Show all of your work** in a neat and orderly fashion to receive maximum partial credit. Failure to include the work required to obtain your solution will result in a score of zero for a given problem. This will be the grading policy even if you have the correct answer.
5. The estimated time to complete this exam is **50 minutes**.
6. This test is an individual effort. Evidence of cheating will result in a score of **zero**.

Contents:

1. This exam packet contains **5 problems**.
2. The last page is **intentionally left blank**. Use this page if additional space is needed for any of the five exam problems.
3. There are a total of **100 points**.

Comments

1. The exam problems are in no particular order.
2. **Do not spend too much time on any particular problem.** If you get stuck on a problem, **move on to the next problem**.
3. The exam is difficult. **Do not let yourself get discouraged!**
## TABLE 4-1
Fourier Transform Theorems<sup>a</sup>

<table>
<thead>
<tr>
<th>Name of Theorem</th>
<th>Symbolism 1</th>
<th>Symbolism 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Superposition ($a_1$ and $a_2$ and arbitrary constants)</td>
<td>$a_1 x_1(t) + a_2 x_2(t)$</td>
<td>$a_1 X_1(f) + a_2 X_2(f)$</td>
</tr>
<tr>
<td>2. Time delay</td>
<td>$x(t - t_0)$</td>
<td>$X(f) e^{-j2\pi f t_0}$</td>
</tr>
<tr>
<td>3a. Scale change</td>
<td>$x(at)$</td>
<td>$</td>
</tr>
<tr>
<td>3b. Time reversal</td>
<td>$x(-t)$</td>
<td>$X(-f) = X^* (f)$</td>
</tr>
<tr>
<td>4. Duality</td>
<td>$X(t)$</td>
<td>$x(-f)$</td>
</tr>
<tr>
<td>5a. Frequency translation</td>
<td>$x(t)e^{j\omega_0 t}$</td>
<td>$X(f - f_0)$</td>
</tr>
<tr>
<td>5b. Modulation</td>
<td>$x(t) \cos \omega_0 t$</td>
<td>$\frac{1}{2} X(f - f_0) + \frac{1}{2} X(f + f_0)$</td>
</tr>
<tr>
<td>6. Differentiation</td>
<td>$\frac{d^n x(t)}{dt^n}$</td>
<td>$(j2\pi f)^n X(f)$</td>
</tr>
<tr>
<td>7. Integration</td>
<td>$\int_{-\infty}^{t} x(t') , dt'$</td>
<td>$(j2\pi f)^{-1} X(f) + \frac{1}{2} X(0) \delta(f)$</td>
</tr>
<tr>
<td>8. Convolution</td>
<td>$\int_{-\infty}^{\infty} x_1(t - t') x_2(t') , dt'$</td>
<td>$X_1(f) X_2(f)$</td>
</tr>
<tr>
<td></td>
<td>$= \int_{-\infty}^{\infty} x_1(t') x_2(t - t') , dt'$</td>
<td></td>
</tr>
<tr>
<td>9. Multiplication</td>
<td>$x_1(t) x_2(t)$</td>
<td>$\int_{-\infty}^{\infty} X_1(f - f') X_2(f') , df'$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$= \int_{-\infty}^{\infty} X_1(f') X_2(f - f') , df'$</td>
</tr>
</tbody>
</table>

<sup>a</sup>$\omega_0 = 2\pi f_0$; $x(t)$ is assumed to be real in 3b.
### TABLE 4-2
Fourier Transform Pairs

<table>
<thead>
<tr>
<th>Pair Number</th>
<th>( x(t) )</th>
<th>( X(f) )</th>
<th>Comments on Derivation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( \Pi \left( \frac{t}{\tau} \right) )</td>
<td>( \tau \text{sinc} , \tau f )</td>
<td>Direct evaluation</td>
</tr>
<tr>
<td>2.</td>
<td>( 2W \text{sinc} , 2Wt )</td>
<td>( \Pi \left( \frac{f}{2W} \right) )</td>
<td>Duality with pair 1, Example 4-7</td>
</tr>
<tr>
<td>3.</td>
<td>( \Lambda \left( \frac{t}{\tau} \right) )</td>
<td>( \tau \text{sinc}^2 , \tau f )</td>
<td>Convolution using pair 1</td>
</tr>
<tr>
<td>4.</td>
<td>( \exp(-\alpha t)u(t) ), ( \alpha &gt; 0 )</td>
<td>( \frac{1}{\alpha + j2\pi f} )</td>
<td>Direct evaluation</td>
</tr>
<tr>
<td>5.</td>
<td>( t \exp(-\alpha t)u(t) ), ( \alpha &gt; 0 )</td>
<td>( \frac{1}{(\alpha + j2\pi f)^2} )</td>
<td>Differentiation of pair 4 with respect to ( \alpha )</td>
</tr>
<tr>
<td>6.</td>
<td>( \exp(-\alpha</td>
<td>t</td>
<td>) ), ( \alpha &gt; 0 )</td>
</tr>
<tr>
<td>7.</td>
<td>( e^{-\pi t^2} )</td>
<td>( \tau e^{-\pi \rho^2} )</td>
<td>Direct evaluation</td>
</tr>
<tr>
<td>8.</td>
<td>( \delta(t) )</td>
<td>1</td>
<td>Example 4-9</td>
</tr>
<tr>
<td>9.</td>
<td>( \delta(t - t_0) )</td>
<td>( \exp(-j2\pi ft_0) )</td>
<td>Duality with pair 7</td>
</tr>
<tr>
<td>10.</td>
<td>( \exp(j2\pi ft) )</td>
<td>( \delta(f - f_0) )</td>
<td>Shift and pair 7</td>
</tr>
<tr>
<td>11.</td>
<td>( \cos 2\pi ft )</td>
<td>( \frac{1}{2j} \delta(f - f_0) + \frac{1}{2} \delta(f + f_0) )</td>
<td>Duality with pair 9</td>
</tr>
<tr>
<td>12.</td>
<td>( \sin 2\pi ft )</td>
<td>( \frac{1}{2j} \delta(f - f_0) - \frac{1}{2j} \delta(f + f_0) )</td>
<td>Exponential representation of cos and sin and pair 10</td>
</tr>
<tr>
<td>13.</td>
<td>( u(t) )</td>
<td>( (j2\pi f)^{-1} + \frac{1}{2} \delta(f) )</td>
<td>Integration and pair 7</td>
</tr>
<tr>
<td>14.</td>
<td>( \text{sgn} , t )</td>
<td>( (j\pi f)^{-1} )</td>
<td>Pair 8 and pair 13 with superposition</td>
</tr>
<tr>
<td>15.</td>
<td>1</td>
<td>(-j \text{sgn}(f) )</td>
<td>Duality with pair 14</td>
</tr>
<tr>
<td>16.</td>
<td>( \hat{x}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\lambda)}{t - \lambda} , d\lambda )</td>
<td>( -j \text{sgn}(f)X(f) )</td>
<td>Convolution and pair 15</td>
</tr>
<tr>
<td>17.</td>
<td>( \sum_{m=-\infty}^{\infty} \delta(t - mT_s) )</td>
<td>( f_s \sum_{m=-\infty}^{\infty} \delta(f - mf_s) ), ( f_s = T_s^{-1} )</td>
<td>Example 4-10</td>
</tr>
</tbody>
</table>
Problem 1 (25 points). Consider the following circuit. \( x(t) \) is the input voltage, and \( y(t) \) is the output voltage. Let \( L = R = 2 \).

1. Is this a high-pass, band-pass, low-pass, or band-stop filter? Explain your answer.

2. Find the impulse response of this filter, \( h(t) \). Show your work.

\[
H(f) = \frac{R}{R + j\omega L} = \frac{X(f)}{X(f)} = \frac{R/L}{R/L + j\omega} = \frac{R}{R^2 + j\omega}
\]

\[
= \frac{1}{1 + j\omega}
\]

\[
h_R(t) = \mathcal{F}^{-1}\{H(f)\} = e^{-t}u(t)
\]

3. Find the impulse response of the filter if the output is taken across the inductor instead of the resistor.

\[
h_L(t) = s(t) - h_R(t) = s(t) - e^{-t}u(t)
\]
Problem 2 (25 Points). Consider an ideal sampling circuit that has a sampling period given by $T_s = 0.01$ seconds.

1. Will the signal $x(t) = \cos(250\pi t - \pi/4)$ be aliased by this sampler?

\[ f_s = 100 \text{ Hz} \]

\[ \Rightarrow \text{Any freq above 50 Hz is aliased.} \]

Yes, aliased.

2. Will the signal $x(t) = \sin(20\pi t)$ be aliased by this sampler?

\[ f_c = \text{max freq. is 10} \text{ Hz} \]

No, max freq. is 10 Hz.

3. Will the signal $x(t) = \text{sinc}(t)$ be aliased by this sampler?

\[ X(f) = T(f), \text{ max freq. of } X(f) = \frac{1}{2} \text{ Hz} \]

\[ \Rightarrow \text{No aliasing.} \]

4. Will the signal $x(t) = \Lambda(t)$ be aliased by this sampler?

\[ X(f) = \text{sinc}^2(f), \text{ which has max freq. of } \infty \text{ Hz} \]

\[ \Rightarrow \text{This signal will be aliased by any sampler.} \]
Problem 3 (25 points).

1. Sketch the magnitude and phase of the Fourier Transform of the signal \( x(t) = 2 \text{sinc}(t) \sin(20\pi t) \).

\[
x(f) = 2 \pi(f) \ast \left[ \frac{1}{\sqrt{2}} \text{sinc}(f-10) + \frac{1}{\sqrt{2}} \text{sinc}(f+10) \right]
\]

2. Sketch the magnitude and phase of the Fourier Transform of the signal \( x(t) = 4 \pi(2t) \).

\[
x(f) = 2 \text{sinc}(f/2)
\]

3. Sketch the magnitude and phase of the Fourier Transform of the signal \( x(t) = \text{sinc}(t-1) \).

\[
x(f) = \pi(f) e^{-j2\pi f^2}
\]
Problem 4 (25 points). The signal \( x(t) = 1 + \cos(2\pi t) + \sin(4\pi t) \) is input to a LTI system that has an impulse response \( h(t) = 8 \text{sinc}^2(4t) \).

1. Sketch the magnitude and phase of the transfer function for this system.

\[
H(f) = 2 \angle \left( \frac{f}{4} \right)
\]

2. Find \( y(t) \). Simplify your answer as much as possible.

\( X(f) \) has energy at the frequencies 0, 1, and 2 Hertz. The filter scales that energy by factors of 2, 1.5, and 1, respectively. Thus,

\[
y(t) = 2 + \frac{3}{2} \cos(2\pi t) + \sin(4\pi t)
\]
Problem 5. Let the input to the following circuit be $x(t) = \cos(20\pi t)$.

$$X(f) = \frac{1}{2} \delta(f - \omega) + \frac{1}{2} \delta(f + \omega)$$

1. Carefully sketch the magnitude and phase of the Fourier Transform of $a(t)$.
2. Carefully sketch the magnitude and phase of $b(f)$.

1. $$A(f) = X(f) \ast \left[ \frac{1}{\sqrt{2}} \delta(f - 100) \right]$$

$$= \frac{1}{\sqrt{2}} X(f - 100) - \frac{1}{\sqrt{2}} X(f + 100)$$

$$|A(f)|$$

2. $$B(f) = A(f) \cdot H(f)$$

$$|B(f)|$$