1. The p.m.f. for Y = the number of blood donors who must be "typed" before finding one of the O+ donors is as follows:

Y	1	2	3	4
p(y)	.1	.2	.3	.4

a) What is the expected number of typings necessary to find an O+ donor?

$E(x) = 1 \times .1 + 2 \times .2 + 3 \times .3 + 4 \times .4 = 3$

b) What is the variance of this number?

$V(X) = (1-3)^2 .1 + (2-3)^2 .2 + (3-3)^2 .3 + (4-3)^2 .4$

$V(x) = 4 + 1 + 0 + 1$

$V(x) = 6$

-2

c) If the r.v. X = # minutes of break time (for the phlebotomist) = max(0,15 – Y^2), find the expected length of her break.

$E(15-y^2) = \sum(15-y^2)\cdot p = (15-1^2).1 + (15-2^2).2 + (15-3^2).3 + (15-4^2).4$

$= 1.4 + 2.2 + 1.8 + 0 = 5.4$ ✓

2. Sixty percent of maize seeds carry double spikelets; of these, 74% will, in turn, produce an ear with double spikelets. Of those with single spikelets, only 71% produce an ear with double spikelets. We randomly choose 15 maize seeds. If our random variable X is the number of the 15 of any sort that produce an ear with double spikelets,

a) what is the expectation and standard deviation of X? (Hint: you may want to draw a probability tree.)

Prob of double = .6 × .74 + .4 × .71 = .728 ✓

$E(x) = 15(.728) = 10.92$ ✓

$V(x) = 2.9704$

$\sigma(x) = \sqrt{2.9704} = 1.7235$ ✓

b) what is the probability that exactly 10 produce an ear with double spikelets?

Binomdist(15, 10, .728 false)

$n = 15 \quad x = 10 \quad p = .728$

Probability = .18694 ✓

3. Grasshoppers are distributed at random in a large field according to a Poisson distribution with parameter $\alpha = 3$ per square yard.

a) If an environmental engineer and her assistant each independently select one square yard to check, what is the probability that each finds no grasshoppers?

$t = 1 \quad \alpha = 3 \quad x = 0$

$P(0) = \frac{e^{-3} 3^0}{0!} = .049787$ ✓

because independent Prob both find 0 is = (.049787)(.049787) = .00248 ✓

b) How large should the radius R of a circular sampling region be taken so that the probability of finding at least one in the region equals .99?

$1 - .99 = \frac{e^{-3t} \cdot t^0}{0!}$ → Area = πR^2 $\quad t = R$

$\ln(.01) = \ln(e^{-3t})$

$-4.605 = -3t$

$t = 1.53506$

-3

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4. Let X denote the time to failure (in years) of a certain hydraulic component. Suppose the pdf of X is $f(x) = 32/(x+4)^2$ for $x>0$.

a) Verify that $f(x)$ is a legitimate pdf.

to be legit $\int f(x)dx = 1$

$= \int_0^\infty 32/(x+4)^2\, dx$

$= 1$ ∴ is legit

ok

b) Calculate the probability that the time to failure is between two and five years.

$P = \int_2^5 32/(x+4)^2 = 32\left[\frac{(5+4)^{-1}}{-1} - \left(\frac{(2+4)^{-1}}{-1}\right)\right]$

5. The pdf for r.v. Y = the distance of a break from a joint is

$$f(y) = \begin{cases} \frac{1}{24} \cdot y \cdot \left(1 - \left(\frac{y}{12}\right)\right) & 0 \le y \le 12 \\ 0 & \text{else} \end{cases}$$

Find E(Y), E(Y^2), and V(Y).

$E(Y) = \int_0^{12} \frac{1}{24} \cdot y \cdot (1 - y/12) \cdot y\, dy$

$E(Y^2) = \int_0^{12} \frac{1}{24} \cdot y \cdot (1 - y/12) \cdot y^2\, dy$

$V(Y) = (E(Y^2)) - E(Y)^2$

$E(Y) = \frac{1}{24}\left(\frac{y^3}{3} - \frac{y^4}{48}\right)\Big|_0^{12} = 6$

$E(Y^2) = \frac{1}{24}\left(\frac{y^4}{4} - \frac{y^5}{60}\right)\Big|_0^{12} = 43.2$

−2

6. The pdf for X = amount of a trace isotope after a microsecond is:

$$f(x) = \begin{cases} e^{-x} & 0 \le x \\ 0 & \text{else} \end{cases}$$

a) Find the cdf.

cdf $= F(x) = \int f(x)$

$\begin{cases} 0 & x<0 \\ 1 - e^{-x} & x \ge 0 \end{cases}$

−2

b) Find the median (50[th] percentile) amount of isotope after a microsecond.

$.5 = \int_0^x e^{-x}\, dx$

solve for x

$.5 = -e^{-x}\Big|_0^x$

$.5 = -e^{-x} + 1$

$-.5 = -e^{-x}$

$\ln .5 = \ln(e^{-x})$

$-.693 = -x$

$.693 = x$ ✓

7. Four hundred amateur fisherman enter a contest to catch one of 5,000 tagged fish in a lake with 25,000 fish total. If each fisherman is allowed to catch only one fish (which is returned to the lake), and the fisherman do their angling independently,

a) What is the exact distribution of the X = number of fisherman who catch a tagged fish?

N = 400 1 of 5000 with 25000 total p = .2 λ = .2 · 400 = 80

binomdist (X, N, P, False) N = 400, p = .2 ✓ False means not cumulative

b) What is the (approximate) probability that more than 67 fisherman catch a tagged fish?

λ = n·p

X = 67

$P(67) = \frac{e^{-\lambda} \lambda^x}{x!} = \frac{e^{-80} 80^{67}}{67!}$

P(66) = .0623 ✓ 1 − P(66) = more than 67 = .9377

Can't use n·p = 80 > 5

−4

8. There are two machines available for cutting corks intended for use in wine bottles. The first produces corks with diameters that are normally distributed with mean 3 cm and standard deviation .1 cm. The second machine produces corks with diameters that have a normal distribution with mean 3.04 cm and standard deviation .02 cm. Acceptable corks have diameters between 2.9 cm and 3.1 cm. Which machine is more likely to produce an acceptable cork? (Justify your answer.)

1st mach – μ = 3cm σ = .1cm 2nd μ = 3.04 σ = .02cm

three σ = 3 · .1 = .3 3σ = 3 · .02 = .06 ✓

the range is 2.7 − 3.3 the range is 2.98 − 3.1

The 2nd machine b/c its range meets the acceptable parameter. The first mach has more of its values that are not acceptable

9. The inside diameter of a randomly selected piston ring is a random variable with mean value 12 cm and standard deviation .04 cm.

a) State the mean and the standard deviation for the distribution of , for each of the following sample sizes: 4, 16, 64

μ = 12 σ = .04 $\sigma = \frac{\sigma}{\sqrt{n}}$

4 μ = 12 σ = .02

16 μ = 12 σ = .01 ✓

64 μ = 12 σ = .005

b) Which of your answers in a), if any, depend on knowing the distribution *shape* for the inside diameter of a randomly selected piston ring?

neither of the answers depend on knowing the distribution shape ✓

10. There are 40 classes in a class, and it is known that the time to grade a randomly selected exam paper is a random variable with an expected value of 6 min and a standard deviation of 6 min. If grading times are independent and the instructor begins grading at 6:50 and grades continuously, what is the (approximate) probability that he is through grading in time for the 11:00 news report? For the 11:10 sports update?

μ = 6 min σ = 6 min

T = 40 · 6 = 240

X = 250 $z = \frac{250 - 240}{6 \cdot \sqrt{40}} = .1667$

probability for that

−3

X = 260 $z = \frac{260 - 240}{6 \cdot \sqrt{40}} = .33$ Find probability for that!