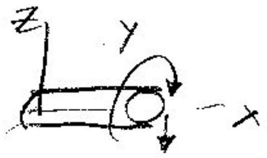
1. A circular shaft of constant diameter, d, is subject to a maximum bending moment, M, and a maximum applied torque, T. If the shaft does NOT rotate, use the Distortion Energy Theory of Failure to show that the minimum shaft diameter is given by:

$$d = \left[\frac{16n}{\pi S_y}\left(4M^2 + 3T^2\right)^{1/2}\right]^{1/3}$$



where, S_y is the yield strength, and n is the safety factor.

30

$$SF = \frac{S_y}{\sigma_e}$$

$$\sigma_x = \frac{M d}{2 I} = \frac{M d}{\frac{2\pi}{64} d^4} = \frac{32 M}{\pi d^3}$$

$$\tau_{xy} = \frac{T \frac{d}{\cancel{2}}}{\left(\frac{\pi}{\cancel{32}_{16}} d^4\right)} = \frac{16 T}{\pi d^3}$$

C

$$\sigma' = \sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + \frac{6\left[\left(\frac{16T}{\pi d^3}\right)^2\right]}{2}}$$

$$\frac{32^2}{\pi^2}\frac{M^2}{d^6} + \frac{3(16^2)T^2}{\pi^2 d^6}$$

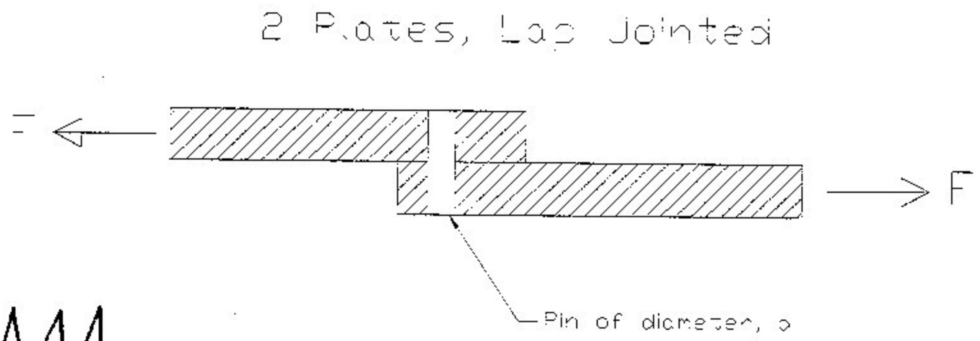
$$\sigma' = \sqrt{\frac{1024 M^2}{\pi^2 d^6} + 768\frac{T^2}{\pi^2 d^6}}$$

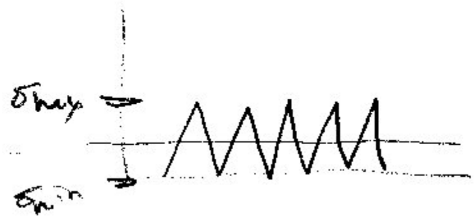
$$n = \frac{S_y}{\frac{16}{\pi d^3}\sqrt{4M^2 + 3T^2}} \Rightarrow n = \frac{\pi d^3 S_y}{16\sqrt{4M^2+3T^2}}$$

C $$\boxed{\sqrt[3]{\frac{16n}{S_y \pi}\sqrt{4M^2+3T^2}} = d}$$

32

2. The lap joint shown below is held together by a single, circular, steel pin of diameter, *d*, and subject to a repeated load, F.





If the corrected endurance limit of the pin is known to be 35% of the ultimate tensile strength, derive an expression for the minimum pin diameter required for a safe design by using the appropriate failure theory. (Your answer should be in terms of safety factor, SF; repeated load, F; and ultimate tensile strength, S_{ut}.)

$S_n = .35\, S_{UT}$ $\quad d\,(SF, F, S_{UT})$

$\sigma_m = \sigma_a$ $\quad \sigma_{max}\ \tau \quad \frac{F}{\frac{\pi d^2}{4}} = \frac{4F}{\pi d^2} \quad \frac{2F}{\pi d^2} = \tau_m = \tau_a$ $\quad$ +16

$\sigma_{a,1} = \sigma_{m,1} = \frac{2F}{\pi d^2} \qquad \sigma_{m,3} = \frac{-2F}{\pi d^2} = \sigma_{a,3}$

Safety Factor Eqn's.

$\frac{\sigma_a'}{S_n} + \frac{\sigma_m'}{S_u} = \frac{1}{SF}$ (Infinite Life) $\qquad \frac{1}{SF} = \frac{\sigma'_{max}}{S_y}$

Since the answer must have a S_{UT} term I will not solve using the static failure theory but the infinite life theory.

$$\sigma_m' = \sqrt{\left(\frac{2F}{\pi d^2}\right)^2 + \left(\frac{-2F}{\pi d^2}\right)^2 + \left(\frac{2F}{\pi d^2}\right)^2} = \sqrt{3}\,\frac{2F}{\pi d^2} = \sigma_m' = \sigma_a'$$

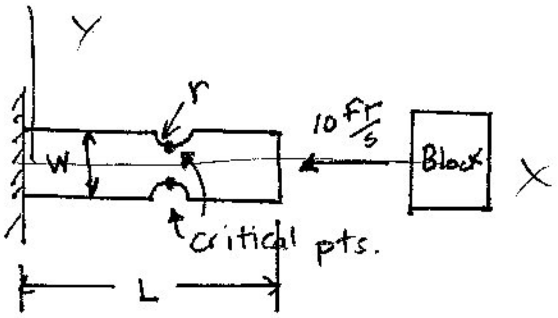
+16

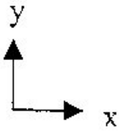
$$\frac{\frac{2\sqrt{3}\,F}{\pi d^2}}{.35\, S_{UT}} + \frac{\frac{2\sqrt{3}F}{\pi d^2}}{S_{UT}} = \frac{1}{SF} \Rightarrow \left(\frac{2\sqrt{3}\,F}{.35\pi d^2 S_{UT}} + \frac{2\sqrt{3}F}{\pi d^2 S_{UT}} = \frac{1}{SF}\right)^{-1}$$

$\rightarrow$

Stupid mistake! In Compression !!

3. A notched rectangular bar is constructed of ASTM 40 Gray Cast Iron. The bar is subjected to an impact loading from block that weighs 5 lbs and is traveling at 10 ft/sec. The radius of the semi-circular notch is r=0.10", the width of the bar is w=1.0" and its thickness into the page is, t=0.5".





$S_{UT} = 42.5$ ksi

a) 10

b) 16

$W = 5$
$V_b = 10\frac{ft}{s}$ $\quad L = 10''$

$r = .1''$

$w = 1''$ $\quad t = .5''$ $\quad L = 10''$

Find:

a. What failure theory would you use to evaluate failure?
b. Using the failure theory from part a., compute the safety factor for the critical point on the [redacted] surface of the bar. Does it fail?

+10 a) $\boxed{\text{Modified Mohr}}$

$H = 1$ $\quad r = .1$ $\quad \frac{r}{h} = \frac{.1}{.8} = .125$ $\quad \frac{H}{h} = \frac{1}{.8} = 1.25$

$h = .8$

$\sigma = K_t \frac{P}{A}$ $\quad$ From chart $K_t = 2.3$ +3

Now we need equivalent force of block when impacts

$E = 15e6$ psi selected min from table

$$F_e = A\sqrt{\frac{2UE}{AL}} = .5\sqrt{\frac{2\frac{1}{2}mv^2E}{.5(10)}} = .5\sqrt{\frac{\frac{5}{\frac{32.2}{12}}(120)^2\ 15e6}{.5(10)}}$$

O.K. +2 $\quad$ +8

$F_e = 11821.656$ lb

$$\sigma_x = \frac{2.3(11821.656)\text{ lb}}{.8''(.5)''} = -67.9745 \text{ ksi}$$

— compression

Since no applied torque $\sigma_1 = \sigma_x$
$\sigma_2 = 0$
$\sigma_3 = 0$

+1