

**EE640 STOCHASTIC SYSTEMS
 SPRING 2008
 COMPUTER PROJECT 1**

PART C: DETECTION and DISCRIMINATION(updated 4-22-08)

1. Fisher Discriminant: Perform a fisher discriminant measure¹ (Rayleigh quotient) on each of the target/clutter vector pairs:

$$t_{li}, c_{li} \quad i = 1, 2, 3$$

$$J_i = \frac{(\mu_{li} - \mu_{ci})^2}{\sigma_{li}^2 + \sigma_{ci}^2} \quad i = 1, 2, 3$$

(C-1)

$$\text{where } \sigma_{li}^2 = \frac{1}{N-1} \sum_{m=1}^N (t_{li}[m] - \mu_{li})^2$$

$$\sigma_{ci}^2 = \frac{1}{N-1} \sum_{m=1}^N (c_{li}[m] - \mu_{ci})^2$$

2. Mathematically and in words describe a MLR test for 3 correlated random variables. **Assume equal covariances and reduce to the linear form.** Using vector pairs (target/clutter pair t_{li}, c_{li}) from problem 1, with the highest J_i value first, form a MLR discriminator for one, two and three r.v.s.
- a. Use the average of the target and clutter covariance matrices to get a common Covariance Matrix. For each of these discriminators, plot the target data response and clutter data response² on the same graph. Estimate minimum probability of error from graphs and plot the three MPE values. Assume $P(\text{target}) = P(\text{clutter}) = 1/2$.
- b. Repeat process in 2(a) for the independent data vectors but don't make any assumptions about the Covariance matrices being diagonal (i.e., keep in quadratic form). Only do the 3 variable case.
3. Form a M=64 long detection filter from a segment of a noise sequence and correlate with the original sequence. Plot correlation and note peak location. At peak location in output correlation determine the performance measures¹, Signal-to-Noise Ratio, $\text{SNR} = \text{peak}^2 / \text{variance}$ and also Peak-to-Average Correlation Energy, $\text{PACE} = \text{peak}^2 / (\text{average squared value of output correlation})$. The correlation is given in terms of convolution as:

(C-2)

$$y[n] = x[n] \otimes h^*[-n]$$

where in the frequency domain this is

$$Y[k] = X[k]H^*[k] \quad (C-3)$$

The entire process is implemented in MATLAB by `y=ifft(fft(x).*conj(fft(h)))`. Solve this problem for the following two cases:

The signal sequence ($x[n]$) is \underline{b}_{binary} and the filter is

$$\underline{h}[n] = \begin{cases} \underline{b}_{binary}[n + m_1] & \text{for } n = 1, 2, \dots, M \\ 0 & \text{for } n = (M + 1), (M + 2), \dots, N \end{cases} \quad (C-4)$$

where m_1 is chosen arbitrarily between 1 and $N-M$.

- b. Repeat 3a for ($x[n]$) is $\underline{s}_{intensity}$.

4. ENCODING INFORMATION INTO THE COLOR NOISE OF A RGB IMAGE

Your objective is to encode and decode a pattern hidden in the color noise of an RGB image.

Display the images of each step and describe with a few words their dimensions and contents.

Assume the host image is a 24 bit per pixel BMP formatted image with dimensions $M \times N$ pixels.

You will input and separate this image into three $M \times N$ 8 bit component images. You can use the following MATLAB code to do this

```
A_bmp=double(imread('example.bmp')); % load example.bmp image
A_r=A_bmp(:,:,1);
A_g=A_bmp(:,:,2);
A_b=A_bmp(:,:,3);
```

where the matrix $A = \{ A_r, A_g, A_b \}$

Define a set of 6 signature tiles. For each Red, Green and Blue image, there is a target tile and a clutter tile. We will refer to these tile pairs as $\{T_r, C_r\}$, $\{T_g, C_g\}$ and $\{T_b, C_b\}$. The dimensions of the tiles are $M_t \times N_t$ where M and N are integer multiples of M_t and N_t , respectively. These tiles are uniform noise with their pixel ranges from $-d$ to $+d$, zero mean and all pixels are iid. The message image B is binary (you can use MS paint to create this and then input using `imread`) and of dimensions $(M/M_t) \times (N/N_t)$.

Create the noise images B_r , B_g and B_b by using a Kronecker multiplication, “ \otimes ”, such that

$$B_r = B \otimes T_r + (1-B) \otimes C_r$$

$$B_g = B \otimes T_g + (1-B) \otimes C_g$$

$$B_b = B \otimes T_b + (1-B) \otimes C_b$$

Add the noise to the host image components (be sure to clip or limit sum values between 0 and 255) such that

$$E_r = A_r + B_r$$

$$E_g = A_g + B_g$$

$$E_b = A_b + B_b$$

You should choose “ d ” to minimize the apparent noise corruption of A . Try $d=5$ for starters but see what the minimum value should approximately be.

Combine E_r , E_g , and E_b to form an RGB image. You can use `imwrite` to store it and email me a copy of A and E to look at via email or get a color print out and include in your project.

Now you need to decode the image.

Input E and separate into E_r , E_g and E_b . Correlate with the appropriate signature filters. Indicate your filter design considerations. Show individual correlation and the sum of all three correlation planes. You may have to threshold and binarize these to bring out the peak locations (you can use negative images to save ink) and also plot final correlation in 3-D mesh so the correlation plane topology can be seen.

SUGGESTED PROJECT LAYOUT (1)

(Note: this is just a suggested layout, it does not guarantee an A grade)

1. Title Page
2. Introduction and description of project. (Turn in with Part C)
3. Project 1A. Discussion of Synthesis (no more than 1 page).
4. Project 1B. Discussion of Analysis (no more than 1 page)
5. Project 1C. Discussion of Detection and Discrimination
 - Explain and present Fisher ratios.
 - Derive MLR tests for 1,2 and 3 variables.
 - Plot results for 1 variable. Plot results for 2 variables.
 - Plot results for 3 variables.
 - Plot out and describe problem 1C (3).
 - Plot out and describe details of problem 1C (4).
6. Conclusions and References.

Appendix: Source Code.

References

1. B.V.K. Vijaya Kumar and L.G. Hassebrook, "Performance Measures for Correlation Filters," *Applied Optics*, **29**, 2997-3006, (July 1990).
2. L. G. Hassebrook, B.V.K. Vijaya Kumar and L. Hostetler, "Linear Phase Coefficient Composite Filter Banks for Distortion-Invariant Optical Pattern Recognition," *Optical Engineering*, **29**, 1033-1043, (Sept. 1990).