EE640 STOCHASTIC SYSTEMS
SPRING 2008
COMPUTER PROJECT 1B

PART B: ANALYSIS(updated 4-23-08)

Use the $\underline{u}_i$ and $\underline{g}_i$ Nx1 lexicographical vectors from Project 1A for the following problems.

1. (a) Generate independent identically distributed test vectors. The target vectors are:

$$\begin{aligned} \underline{t}_{I1} &= \underline{g}_1 + 3 \\ \underline{t}_{I2} &= \underline{g}_2 + 3 \\ \underline{t}_{I3} &= \underline{g}_3 + 3 \end{aligned} \qquad \text{(B-1a)}$$

and the clutter vectors are:

$$\begin{aligned} \underline{c}_{I1} &= \underline{g}_4 + 1 \\ \underline{c}_{I2} &= \underline{g}_5 + 1 \\ \underline{c}_{I3} &= \underline{g}_6 + 1 \end{aligned} \qquad \text{(B-1b)}$$

(b) Generate correlated test vectors. Three vectors will represent target class data and three vectors will represent clutter class data. The 3 target vectors are:

$$\begin{aligned} \underline{t}_1 &= 4\underline{g}_1 + 2\underline{g}_2 + \underline{g}_3 + 34 \\ \underline{t}_2 &= \underline{g}_1 + 4\underline{g}_2 + 2\underline{g}_3 + 26 \\ \underline{t}_3 &= 2\underline{g}_1 + \underline{g}_2 + 4\underline{g}_3 + 38 \end{aligned} \qquad \text{(B-2a)}$$

and the clutter vectors are:

$$\begin{aligned} \underline{c}_1 &= 4\underline{g}_4 + 2\underline{g}_5 + \underline{g}_6 + 6 \\ \underline{c}_2 &= \underline{g}_4 + 4\underline{g}_5 + 2\underline{g}_6 + 12 \\ \underline{c}_3 &= 2\underline{g}_4 + \underline{g}_5 + 4\underline{g}_6 + 17 \end{aligned} \qquad \text{(B-2b)}$$

2. Histogram: Design a program, or use the MATLAB function hist.m, that will estimate the histogram of random image intensity values. Have the program use specified *M* bin intervals. Run the program and plot for:

$$
\begin{array}{lcl}
PlotB\text{-}1 & \underline{u}_1,\ \underline{g}_1 & histogram \\
PlotB\text{-}2 & \underline{s}_1,\underline{s}_2,\underline{s}_3,\underline{s}_4,\underline{s}_5 & histogram \\
PlotB\text{-}3 & \underline{t}_{I1},\ \underline{c}_{I1} & histogram \\
PlotB\text{-}4 & \underline{t}_{I2},\ \underline{c}_{I2} & histogram \\
PlotB\text{-}5 & \underline{t}_{I3},\ \underline{c}_{I3} & histogram \\
PlotB-6 & \underline{s}_{intensity} & histogram
\end{array}
$$

3. Covariance estimate: Estimate covariance matrices (3x3)

K_t from $\underline{t}_1,\underline{t}_2,\underline{t}_3$
K_c from $\underline{c}_1,\underline{c}_2,\underline{c}_3$
K_{tI} from $\underline{t}_{I1},\underline{t}_{I2},\underline{t}_{I3}$
K_{cI} from $\underline{c}_{I1},\underline{c}_{I2},\underline{c}_{I3}$

such that:

$$K(m,n)=\frac{1}{(N-1)}(\underline{x}_m-\underline{\mu}_m)^T(\underline{x}_n-\underline{\mu}_n) \tag{B-3}$$

where $\underline{\mu}_m$ is an Nx1 vector with all elements equal to the mean value of the vector $\underline{x}_m$ and $K(m,n)$ is the mth, nth element of the matrix K.

4. Estimate mean vectors (3x1) from

$$\underline{t}_{I1},\underline{t}_{I2},\underline{t}_{I3} \tag{B-4}$$

such that

$$\underline{\mu}_t=\begin{bmatrix}\mu_{t,1}\\ \mu_{t,2}\\ \mu_{t,3}\end{bmatrix} \tag{B-5}$$

where

$$\mu_{t,i}=\frac{1}{N}\sum_{m=1}^{N}\underline{t}_{I,i}[m] \tag{B-6}$$

likewise for clutter, use

$$\underline{c}_{I1},\underline{c}_{I2},\underline{c}_{I3} \tag{B-7}$$

to generate

$$\underline{\mu}_c=\begin{bmatrix}\mu_{c,1}\\ \mu_{c,2}\\ \mu_{c,3}\end{bmatrix} \tag{B-8}$$

5. Use *one* of the target images and corresponding control noise image from Project 1.A.2. Find the histogram of the deterministic image intensity and also find the histogram of the associated control noise intensity. Are these histograms different? Why?

6. Determine the peak element locations, the centroid element locations of the histograms of $\underline{b}_{binary}$ and $\underline{s}_{intensity}$. The centroid is determined by "simulating" a pdf. For example, let x[n] be a sequence and you want to approximate E{x[n]}. Let h(x) be the histogram of x[n]. First, form a pseudo pdf as

$$f_x(x) = \frac{h(x)}{\sum_{m=1}^{M} h[m]} \quad \text{(B-9)}$$

where m is the bin number of a total M bins in the histogram. The value of $h(x)$ returns the bin value that contains the value of x. Let another sequence $x_c[m]$ for $m=1,2,\ldots M$ be the center locations of each bin in $f_x(x)$. Then the centroid is then

$$\mu_x = \sum_{m=1}^{M} x_c[m] f_x(x_c[m]) \quad \text{(B-10)}$$

Determine the time averages of $\underline{b}_{binary}$ and $\underline{s}_{intensity}$ and compare with the centroid averages. They should be close.

Optional Method: A conceptually easier technique for implementing the centroid is the following: The approach will first get the fractional value of the estimated bin number. Then that value is linearly mapped to the x dimension. Given your bin numbers for the histograms $h[m]$ are equally spaced and vary from 1 to M. We can map bin values to signal values with $x_{min}=a*1+b$ and $x_{max}=a*M+b$. So $a=(x_{max}-x_{min})/(M-1)$ and $b=x_{min}-a$. The values x_{max} and x_{min} are the center values associated with the end bins. So we find the centroid

$$f_m(m) = \frac{h[m]}{\sum_{m=1}^{M} h[m]} \quad \text{(B-11)}$$

The centroid is a fractional value

$$\mu_m = \sum_{m=1}^{M} m f_m(m) \quad \text{(B-12)}$$

and the mean of x is then $\mu_x=\mu_m*a+b$.