EE640 4-17-03 (1)

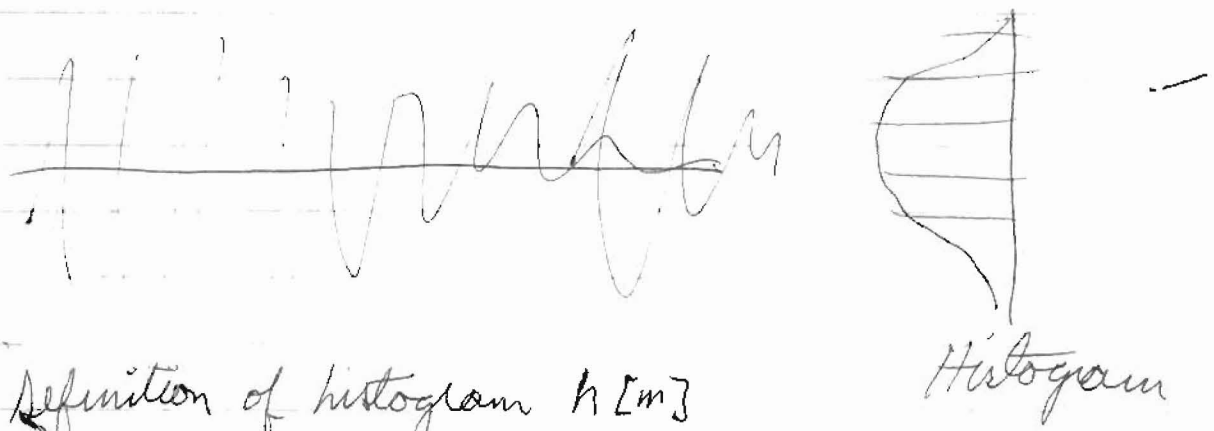
PDF Estimation

given a random process $\tilde{x}(t)$

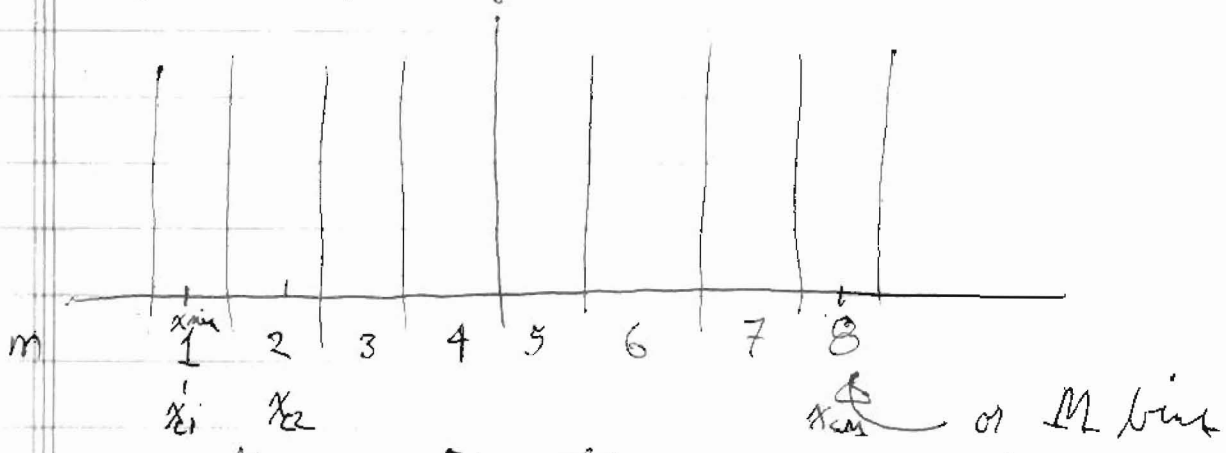
We assume $\tilde{x}(t)$ is ergodic and SSS

Thus we can use time samples to estimate pdf

Let $\tilde{x}_s[n] = \tilde{x}(nT)$ for $n = 1, \cdots, N$



Definition of histogram $h[m]$



$m = 1, 2, \cdots M$ or M bins

Bin centers $x_{cm} = a\,m + b$

~~The bin width~~

Given $x_{c1} = x_{min}$ and $x_{cM} = x_{max}$

and given that all bins are equal width Δx then

$$\Delta x = \frac{x_{cM} - x_{c1}}{(M-1)} = \frac{x_{max} - x_{min}}{(M-1)}$$

The bin boundaries for bin m are

$$x_{L,m} = x_{c,m} - \frac{\Delta x}{2}$$

$$x_{R,m} = x_{c,m} + \frac{\Delta x}{2} \quad \text{s.t.} \quad x_{R,m} \geq x_{L,m}$$

As the number of samples $N \to \infty$

$$h[m] = \int_{x_{L,m}}^{x_{R,m}} f_{\tilde{x}}(x)\, dx = F_x(x_{R,m}) - F_x(x_{L,m})$$

As $\Delta x \to 0$

$$\frac{F_x(x_{R,m}) - F_x(x_{L,m})}{\Delta x} \longrightarrow \frac{\partial F_x(x_{c,m})}{\partial x} = f_x(x_{c,m})$$