# LECTURE NOTES DAY 23
# ECE EE640

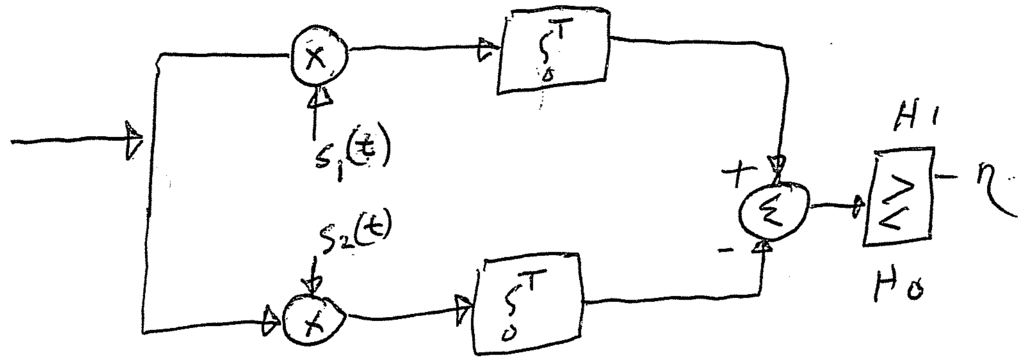
Optimum Detector Filter Banks using correlation
4-12-06

lecture 23

# Optimum Detector Filter Banks

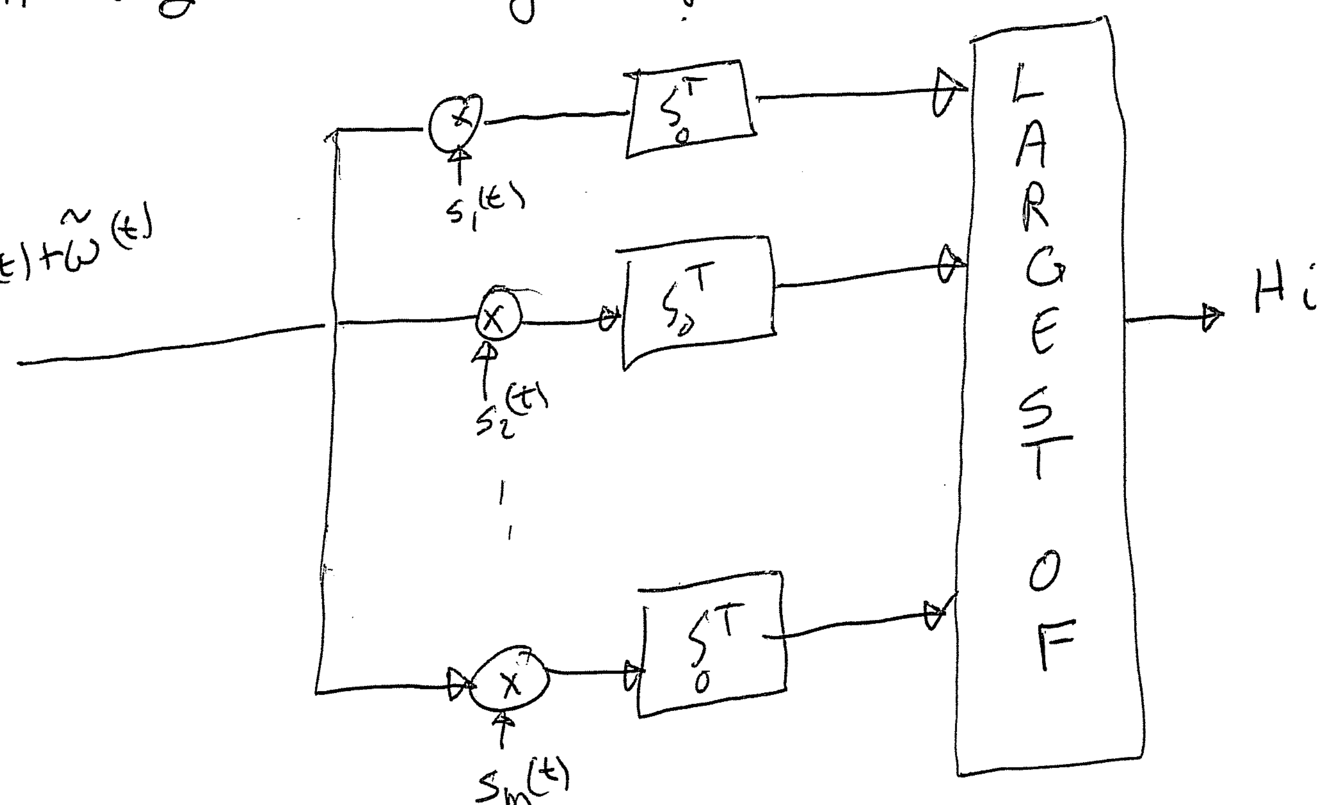
$\tilde{x}(t) = s_i(t) + \tilde{w}(t)$

Binary detector




M-ary or "Largest of" correlator

$\tilde{x}(t) = s_i(t) + \tilde{w}(t)$



Let $\tilde{x}(t) = p(t) + \underbrace{\tilde{\omega}(t)}_{\text{WGN with zero mean}}$

$$\underset{\text{AWGN}}{\tilde{x}(t)} \longrightarrow \boxed{h(t)} \xrightarrow{\text{ACGN}} \tilde{y}(t) \quad \text{where } S_{yy}(\omega) = \frac{N_0}{2} |H(\omega)|^2$$

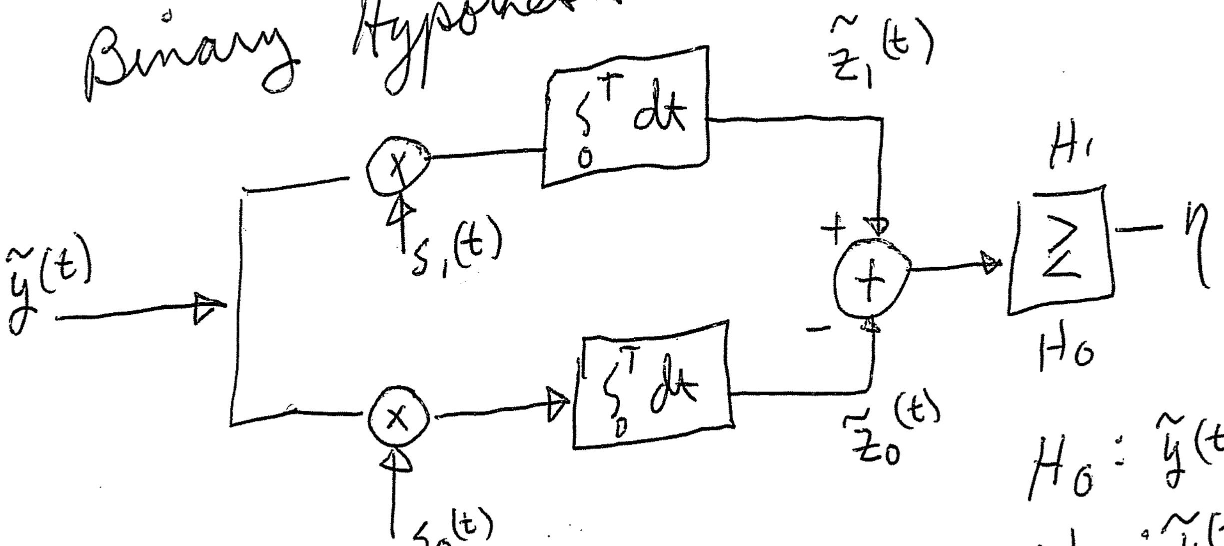
$\tilde{x}(t) = p_0(t) + \tilde{\omega}(t)$

or

$\tilde{x}(t) = p_1(t) + \tilde{\omega}(t)$

<u>Detection</u> ~~and~~

Binary Hypothesis




$H_0 : \tilde{y}(t) = s_0(t) + \tilde{n}(t)$

$H_1 : \tilde{y}(t) = s_1(t) + \tilde{n}(t)$

where $z_i(t) = \int_0^T \tilde{y}(t) s_i(t)\, dt$

given $z_i(t) = \int_0^T \tilde{y}(t)\, s_i(t)\, dt$

Consider

$H_0$: $$z_0(t) = \int_0^T \left(s_0(t) + \tilde{n}(t)\right) s_0(t)\, dt$$

$$= \underbrace{\int_0^T s_0^2(t)\, dt}_{s_{00}} + \underbrace{\int_0^T \tilde{n}(t)\, s_0(t)\, dt}_{\tilde{n}_0}$$

$$z_1(t) = \int_0^T \left(s_0(t) + \tilde{n}(t)\right) s_1(t)\, dt$$

$$= \underbrace{\int_0^T s_0(t)\, s_1(t)\, dt}_{s_{01}} + \underbrace{\int_0^T \tilde{n}(t)\, s_1(t)\, dt}_{\tilde{n}_1}$$

$$\lim_{T \to \infty} \int_0^T \tilde{n}(t) s_i(t)\, dt = E\left\{ \int_0^T \tilde{n}(t)\, s_i(t)\, dt \right\} = \int_0^T E\{\underbrace{\tilde{n}(t)}_{0}\}\, s_i(t)\, dt = 0$$

For $H_1$:

$$z_0(t) = \underbrace{\int_0^T s_1(t)\, s_0(t)\, dt}_{s_{10} = s_{01}} + \tilde{n}_0$$

$$z_1(t) = \underbrace{\int_0^T s_1^2(t)\, dt}_{s_{11}} + \tilde{n}_1$$

Then the decision is made from

$$\tilde{r}(t) = \tilde{z}_1(t) - \tilde{z}_0(t)$$

As

$H_0$: $\tilde{r}(t) = (s_{01} + \tilde{n}_1) - (s_{00} + \tilde{n}_0)$

$H_1$: $\tilde{r}(t) = (s_{11} + \tilde{n}_1) - (s_{01} + \tilde{n}_0)$

Assume we calibrate the signal energy s.t. $s_{11} = s_{00} = E$

then $S_{01} \leq E$

If $s_0(t)$ and $s_1(t)$ are orthogonal then $S_{01} = 0$

$\therefore$

$H_0$: $E\{\hat{r}(t)\} = -S_{00} = -E$

$H_1$: $E\{\tilde{r}(t)\} = S_{11} = +E$