LECTURE NOTES DAY 17
ECE EE640

Detection and Discrimination 3-22-05

Lecture 17

Detection and Discrimination

Consider two signals $\tilde{r}_0$ and $\tilde{r}_1$ where

Hypothesis

case H_0: $\tilde{r} = \tilde{r}_0 = s_0 + \tilde{w}$

and

case H_1: $\tilde{r} = \tilde{r}_1 = s_1 + \tilde{w}$

$\Big\}$ additive noise models

where s_i is a deterministic value and

$\tilde{w} \sim N(0, \sigma^2)$

In binary detection and discrimination we only have 2 cases.

Our goal is to sample $\tilde{r}$ and determine where H_0 or H_1 is present.

We want to make this decision so that we achieve minimum probability of error (MPE).

We know $\quad H_0 : E\{\tilde{r}\} = s_0 \qquad Var\{\tilde{r}\} = \sigma^2$

$\qquad\qquad\quad H_1 : E\{\tilde{r}\} = s_1 \qquad Var\{\tilde{r}\} = \sigma^2$

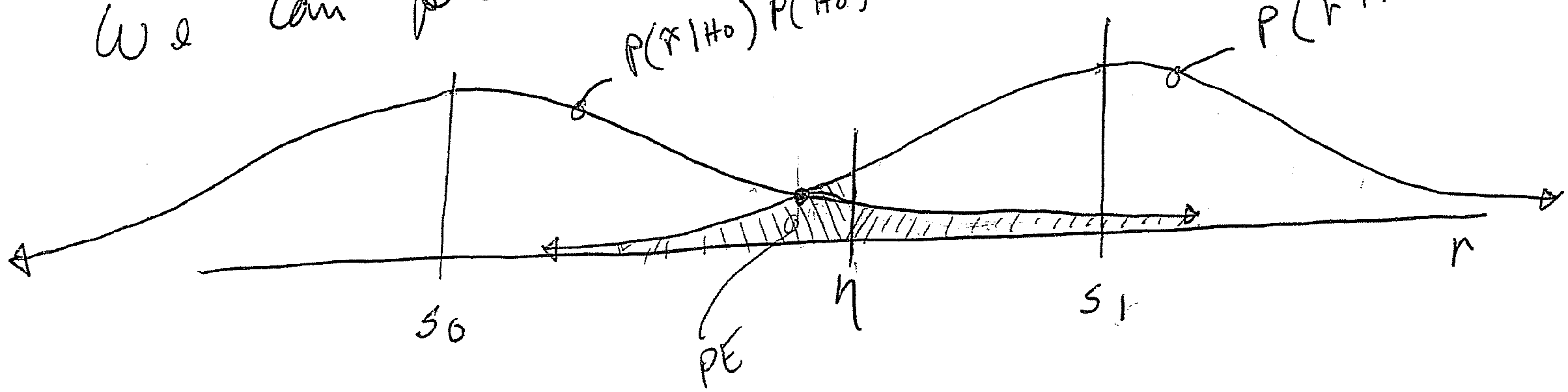
The Prob of $\tilde{r}$ and H_0 is

$$P(\tilde{r}, H_0) = P(\tilde{r} \mid H_0) P(H_0)$$

and for H_1

$$P(\tilde{r}, H_1) = P(\tilde{r} \mid H_1) P(H_1)$$

We can plot ~~these~~ two Prob. s.t.



Decision rule

We decide whether H_0 or H_1 is present by

H_0 if $\tilde{r} < \eta$

H_1 if $\tilde{r} \geq \eta$

There are two Types of error

False alarms and Misses

We choose one case as a target case. Let it be H_1

A False alarm has the probability of

$$P_F = \int_{\eta}^{\infty} P(r \mid H_0) P(H_0)\, dr$$

where a non-Target or sometimes referred to as "clutter" is present but we decide a target present.

A miss has the prob. of

$$P_m = \int_{-\infty}^{\eta} p(r|H_1)\,P(H_1)\,dr$$

where a target is present but we decide a clutter is present.

To minimize the total prob. of error

$$P_e = P_F + P_m$$

we move the threshold to where both cases are equally likely,

or when $p(\tilde{r}|H_0)\,P(H_0) = p(\tilde{r}|H_1)\,P(H_1)$

Because there are only two cases and their probabilities are never negative we can rewrite as a ratio

$$\frac{P(\tilde{r} \mid H_1)\,P(H_1)}{P(\tilde{r} \mid H_0)\,P(H_0)} = 1$$

for equally likely or equal liklihood

The ratio is rewritten as decision process s.t.

$$\frac{P(\tilde{r} \mid H_1)\,P(H_1)}{P(\tilde{r} \mid H_0)\,P(H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} 1$$

Which ever is more likely, that case is decide with MPE

If we don't know the a priori prob. we assume

$$P(H_0) = P(H_1) = 1/2$$

Thus the Maximum likelihood ratio (MLR) can be written as

$$\frac{P(\tilde{r} \mid H_1)}{P(\tilde{r} \mid H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{P(H_0)}{P(H_1)} = 1$$

If $P(H_0) = P(H_1) = \frac{1}{2}$ then

Furthermore, we can take the $\ln(\,)$ of ~~bo~~ both sides without changing the PE because $\ln(\,)$ is a monotonic function s.t.

$$R_{MLR} = \ln\left(\frac{P(\tilde{r} \mid H_1)}{P(\tilde{r} \mid H_0)}\right) \underset{H_0}{\overset{H_1}{\gtrless}} \ln(1) = 0.$$

$$= \ln(P(\tilde{r} \mid H_1)) - \ln(P(\tilde{r} \mid H_0)) \underset{H_0}{\overset{H_1}{\gtrless}} 0$$

if $P(\tilde{r}|H_i) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{1}{2}\frac{(r-s_i)^2}{\sigma^2}\right)$

$$R_{MLR} = \ln\left(\cancel{\frac{1}{\sqrt{2\pi}\,\sigma}}\right) - \frac{1}{2}\frac{(r-s_1)^2}{\sigma^2} - \ln\left(\cancel{\frac{1}{\sqrt{2\pi}\,\sigma}}\right) + \frac{1}{2}\frac{(r-s_0)^2}{\sigma^2} \underset{H_0}{\overset{H_1}{\gtrless}} 0$$

$$= -\frac{1}{2}\left[\frac{r^2 - 2s_1 r + s_1^2}{\sigma^2}\right] + \frac{1}{2}\left[\frac{r^2 - 2s_0 r + s_0^2}{\sigma^2}\right] \gtrless 0$$

$$= \frac{2s_1 r - 2s_0 r - s_1^2 + s_0^2}{2\sigma^2} \underset{H_0}{\overset{H_1}{\gtrless}} 0$$

$$= r \underset{H_0}{\overset{H_1}{\gtrless}} \frac{s_1^2 - s_0^2}{2(s_1 - s_0)} = \eta = \frac{\cancel{(s_1 - s_0)}(s_1 + s_0)}{2\cancel{(s_1 - s_0)}}$$

$$= r \underset{H_0}{\overset{H_1}{\gtrless}} \left(\frac{s_1 + s_0}{2}\right) = \eta = \text{decision boundary}$$