LECTURE NOTES DAY 13 Example
ECE EE640

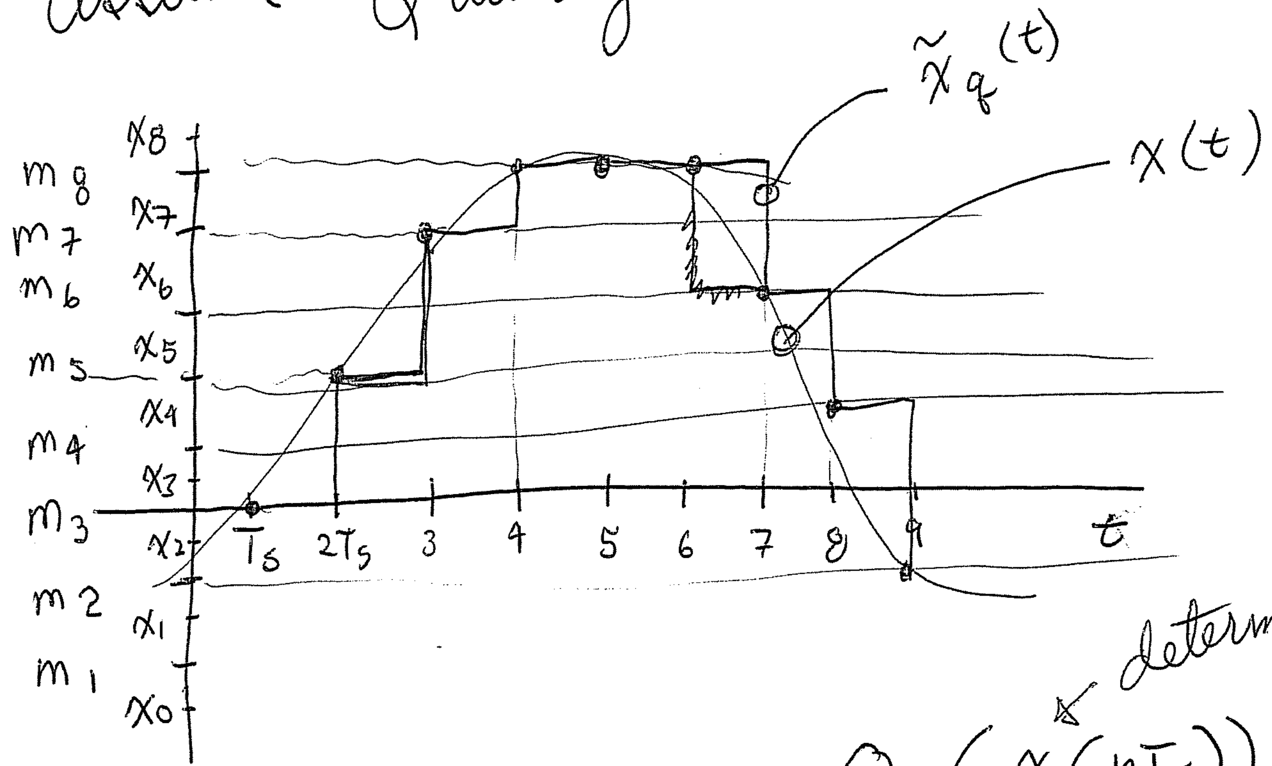
Quantization Noise Model

Lecture 13 example

Quantization Noise

Assume Quantization

$Q=8$



quantization $\tilde{x}_q(t) = Q\left(x(kT_s)\right)$ ← deterministic

let $\Delta = \dfrac{x_8 - x_0}{Q}$

Model quantization as an Additive noise process

$\tilde{x}_q(t) = x(kT_s) + \tilde{\omega}_k$

if $x(t)$ varies enough then the Quantization error can be modeled as noise

Let $\tilde{\omega}_k \sim U(-a, a)$ iid $\forall_k$

$$E\{\tilde{x}_q(t)\} = E\{x(kT_s)\} + E\{\tilde{\omega}_k\}$$

$$= x(kT_s)$$

$$\mathrm{Var}\{\tilde{x}_q(t)\} = E\{(\tilde{x}_q(t) - x(kT_s))^2\}$$

$$= E\{\tilde{\omega}_k^2\}$$

$$= \int_{-\infty}^{\infty} \frac{w^2 \,\mathrm{rect}\left(\frac{w}{2a}\right)}{2a}\, dw = \int_{-a}^{a} \frac{w^2}{2a}\, dw$$

$$= \left.\frac{w^3}{6a}\right|_{-a}^{a} = \frac{a^2}{3}$$

Let $x(t)$ be r.p. $\tilde{x}(t)$

Noise Power

$$N_Q = E\{(\tilde{x}(t) - \tilde{x}_q(t))^2\} = a^2/3$$

Total Signal Power

$$S_Q = E\{(\tilde{x}_q(t))^2\} = \underbrace{E\{\tilde{x}^2(t)\}}_{\text{signal}} + \underbrace{\frac{a^2}{3}}_{\text{noise}}$$

$$\boxed{SNR = \frac{E\{\tilde{x}^2(t)\}}{a^2/3}}$$