EE640 Lecture 10 Central Limit Theorem

We know

$$f_{\tilde{z}}(z) = \int_{-\infty}^{\infty} f_{\tilde{x}}(w)\, f_{\tilde{y}}(z-w)\, dw$$

$$= f_{\tilde{x}}(z) * f_{\tilde{y}}(z)$$

We can regressively apply this relationship to a sum of many i.i.d. r.v.s and obtain what is known as the

Central Limit Theorem

Theorem: Let $\tilde{x}_1, \tilde{x}_2 \cdots \tilde{x}_n$ be iid. r.v.s with cdfs $F_1(x_1) \cdots F_n(x_n)$

We will assume $\mu_{x,k} = 0$ and $Var(\tilde{x}_k) = \sigma_k^2$

Let $\tilde{z}_n \triangleq \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \tilde{x}_i$

$$\lim_{n \to \infty} f_{\tilde{z}_n}(z) = \frac{1}{\sqrt{2\pi}\,\sigma_z} \exp\left\{ -\frac{(z-\mu_z)^2}{2\sigma_z^2} \right\}$$

$$E\{\tilde{z}_n\} = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} E\{\tilde{x}_i\} = 0$$

$$\mathrm{Var}\{\tilde{z}_n\} = E\{\tilde{z}_n^2\} = \frac{1}{n} \sum_{i=1}^{n} E\{\tilde{x}_i^2\}$$

$$= \frac{1}{n} \sum_{i=1}^{n} \sigma_i^2$$

SUMMATIONS OF <u>ANY</u> IND. RVs

APPROACH A GAUSSIAN RV

EX: Central Limit Theorem (CLT)

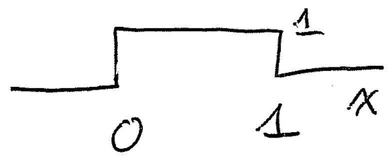
$$\tilde{z} = \tilde{x}_1 + \tilde{x}_2 \; - - - \; \tilde{x}_N$$

and $\tilde{x}_i$ are all independent

$$\xrightarrow{N \to \infty} \quad \tilde{z} \sim N(\mu_z, \sigma_z^2)$$

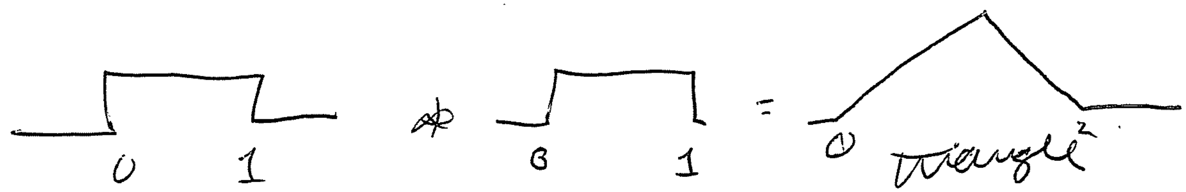
EX: $\hat{z} = \tilde{x}_1 + \tilde{x}_2 + \tilde{x}_3$ where $\hat{x}_i$ are iid

~~with~~ with $f_{\hat{x}_i}(x) =$



let $\hat{\omega} = \hat{x}_1 + \tilde{x}_2$

$f_{\hat{\omega}}(\omega) =$



$\tilde{z} = \tilde{\omega} + \tilde{x}_3$

$f_{\tilde{z}}(z) =$