EE640 STOCHASTIC SYSTEMS

Spring, 2006

COMPUTER PROJECT 1

PART B: ANAYLYSIS (April 5, 2006)

1. Generate independent identically distributed test vectors. The target vectors are:

$$\begin{aligned} \underline{t}_{IID1} &= \underline{g}_1 + 3 \\ \underline{t}_{IID2} &= \underline{g}_2 + 3 \\ \underline{t}_{IID3} &= \underline{g}_3 + 3 \end{aligned} \tag{1}$$

   and the clutter vectors are:

$$\begin{aligned} \underline{c}_{IID1} &= \underline{g}_4 + 1 \\ \underline{c}_{IID2} &= \underline{g}_5 + 1 \\ \underline{c}_{IID3} &= \underline{g}_6 + 1 \end{aligned} \tag{2}$$

2. Histogram: Design a program, or use the MATLAB function hist.m, that will estimate the histogram of an $N \times 1$ vector of random numbers. Have the program use specified M bin intervals. Run the program and plot for:

PlotB-1	$\underline{u}_1$, $\underline{g}_1$	histogram
PlotB-2	$\underline{s}_1$, $\underline{s}_2$, $\underline{s}_3$, $\underline{s}_4$, $\underline{s}_5$	histogram
PlotB-1	$\underline{t}_{IID1}$, $\underline{c}_{IID1}$	histogram
PlotB-1	$\underline{t}_{IID2}$, $\underline{c}_{IID2}$	histogram
PlotB-1	$\underline{t}_{IID3}$, $\underline{c}_{IID3}$	histogram
PlotB-1	$\underline{s}_{intensity}$	histogram

3. Perform tasks 2 and 3 from Project 1S and show graphs.

4. Covariance estimate: estimate covariance matrices (3×3)

   K_{tIID} from $\underline{t}_{IID1}, \underline{t}_{IID2}, \underline{t}_{IID3}$

   K_{cIID} from $\underline{c}_{IID1}, \underline{c}_{IID2}, \underline{c}_{IID3}$

   (In some context, this is called the correlation matrix) i.e.

$$K(m,n) = \frac{1}{(N-1)} \left(\underline{x}_m - \underline{\mu}_m\right)^T \left(\underline{x}_n - \underline{\mu}_n\right) \tag{3}$$

   where $\underline{\mu}_m$ is an $N \times 1$ vector with all elements equal to the mean value of the vector $\underline{x}_m$ and $K(m,n)$ is the m^{th}, n^{th} element of the matrix K.

5. Estimate mean vectors (3×1) from

$$\underline{t}_{IID1}, \underline{t}_{IID2}, \underline{t}_{IID3}, \tag{4}$$

such that

$$\underline{\mu}_t = \begin{bmatrix} \mu_{t,1} \\ \mu_{t,2} \\ \mu_{t,3} \end{bmatrix} \tag{5}$$

likewise for clutter, use

$$\underline{c}_{IID1}, \underline{c}_{IID2}, \underline{c}_{IID3}, \tag{6}$$

to generate

$$\underline{\mu}_c = \begin{bmatrix} \mu_{c,1} \\ \mu_{c,2} \\ \mu_{c,3} \end{bmatrix} \tag{7}$$

6. Repeat step 4 and 5 for the noisy image responses defined in item 6 of project 1A. The responses should be from type 0 LPCCF, order $k = 1, 2$ *and* 3 filters. Note that lexicographic form is used such that the covariance matrix is 3×3.

7. Determine the peak element locations, the centroid element locations of the histograms of $\underline{b}_{binary}$ and $\underline{s}_{intensity}$. The centroid is determined by "simulating" a pdf. For example, lt $x[n]$ be a sequence and you want to approximate $E\{x[n]\}$. Let $h(x)$ be the histogram of $x[n]$. First, form a pseudo pdf as

$$f_x(x) = \frac{h(x)}{\sum\limits_{m=1}^{M} h[m]} \tag{8}$$

where m is the bin number of a total M bins in the histogram. The value of $h(x)$ returns the bin value that contain the value of x. The centroid is then

$$\mu_x = \sum_{n=1}^{N} x[n] f_x(x[n]) \tag{9}$$

Determine the time averages of $\underline{b}_{binary}$ and $\underline{s}_{intensity}$ and compare with the centroid averages. The should be close.

Optional Method: A conceptually easier technique for implementing the centroid is the following: The approach will first get the fractional value of the estimated bin number. Then that value is linearly mapped to the x dimension. Given your bin numbers for the histograms $h[m]$ are equally spaced and vary from 1 to M. We can map bin values to signal values with $x_{min} = a * 1 + b$ and $x_{max} = a * M + b$. So $a = (x_{max} - x_{min})/(M - 1)$ and $b = x_{min} - a$. The value x_{max} and x_{min} are the center values associated with the end bins. So we find the centroid

$$f_m(m) = \frac{h[m]}{\sum\limits_{m=1}^{M} h[m]} \tag{10}$$

The centroid is a fractional value

$$\mu_m = \sum_{m=1}^{M} m f_m(m) \tag{11}$$

and the mean of x is then $\mu_x = \mu_m * a + b$.