

# EE640 STOCHASTIC SYSTEMS

Spring, 2006

## COMPUTER PROJECT 1

### PART A: SYNTHESIS (March 21, 2006)

Let  $N = 512$ :

1. Uniform pseudo-random numbers. Generate 6 random vectors, each with a different seed. The vectors are all  $N^2 \times 1$  where each element is uniformly distributed between 0 and 1. Each element is independent from the others. Mathematically refer to the vectors as:

$$\underline{u}_1, \underline{u}_2, \underline{u}_3, \underline{u}_4, \underline{u}_5, \underline{u}_6, \quad (1)$$

2. Prove the parametric transformation equations for converting from a uniform distribution to a gaussian distribution are correct.

$$\underline{g}_i[2i + 1] = \sqrt{-2 \ln \underline{u}_i[2n + 1]} \cos 2\pi \underline{u}_i[2n + 2] \quad (2)$$

$$\underline{g}_i[2i + 2] = \sqrt{-2 \ln \underline{u}_i[2n + 1]} \sin 2\pi \underline{u}_i[2n + 2] \quad (3)$$

where  $n = 0, 1, 2 \dots (N^2/2 - 1)$

3. Generate six  $N^2 \times 1$  gaussian random vectors from the associated vectors in part (1). Use the transformation developed in (2). Generate them with a 0 mean and unity variance and store as you did in (1). Refer to them as

$$\underline{g}_1, \underline{g}_2, \underline{g}_3, \underline{g}_4, \underline{g}_5, \underline{g}_6, \quad (4)$$

4. Linear combinations of r.v.s. Generate five  $N^2 \times 1$  vectors such that

$$\begin{aligned} \underline{s}_1 &= \underline{u}_1 + \underline{u}_2 \\ \underline{s}_2 &= \underline{u}_1 + \underline{u}_2 + \underline{u}_3 \\ \underline{s}_3 &= \underline{u}_1 + \underline{u}_2 + \underline{u}_3 + \underline{u}_4 \\ \underline{s}_4 &= \underline{u}_1 + \underline{u}_2 + \underline{u}_3 + \underline{u}_4 + \underline{u}_5 \\ \underline{s}_5 &= \underline{u}_1 + \underline{u}_2 + \underline{u}_3 + \underline{u}_4 + \underline{u}_5 + \underline{u}_6 \end{aligned} \quad (5)$$

5. Change  $g_i$  from lexicographical form to 2D matrix  $G_i$ . Put the first  $N$  elements in  $g_i$  to the first row of  $G_i$ , the  $N - 2N$  elements in  $g_i$  to the second row of  $G_i$  and so on. The size of  $g_i$  is  $N^2 \times 1$  and the size of  $G_i$  is  $N \times N$ .
6. Letting  $N = 128$ : Use the 12 training images from Project 1-S: Supplemental "SYNTHESIS OF DETERMINISTIC SYSTEM" and add zero mean white Gaussian Noise to them such that the Signal to Noise Ratio is 0 dB. You can use `randn()` to generate the noise for the 6 target and 6 clutter images. Add them to the images and subplot 12 noisy images.

7. Generate two more 1-D vectors, a pseudo-random binary (i.e., bipolar) sequence and pseudo-random intensity sequence such that:

$$\underline{b}_{binary}[n] = \begin{cases} 1 & \text{for } \underline{u}_1[n] \geq 0.5, \\ -1 & \text{for } \underline{u}_1[n] < 0.5. \end{cases} \quad (6)$$

and

$$\underline{s}_{intensity}[n] = \left(\underline{g}_1[n]\right)^2 \quad (7)$$

where  $n = 1, 2, \dots, N^2$ . We will use these to conduct some signal processing experiments.