Lecture 20: FIR Design

Least Squared Time Sample, Frequency Sample

Design Spec.

Design of FIR filter

$$h[n] = \sum_{k=0}^{m-1} b_k \times [n-k] \quad \text{or} \quad y[n] = \sum_{k=0}^{m-1} h[k] \times [n-k]$$

$$H(z) = \sum_{k=0}^{m-1} h[k] z^{-n}$$
An FIR has linear phase if it is
Symmetric \( h[n] = h[M-n] \)

or
Anti Symmetric \( h[n] = -h[M-n] \)

In the \( z \)-domain, these properties are
\[
2^{-(M-1)} H(\frac{1}{z^{-1}}) = \pm H(z)
\]

The above implies that the roots of \( H(z) \) are
equal to the roots of \( H(\frac{1}{z^{-1}}) \)

ie \( z \), an zero of \( H(z) \) the \( \frac{1}{z} \), is zero of \( H(\frac{1}{z^{-1}}) \)
Heart Square design (time sample)

Ex: Desired frequency response is

\[ H_{LP}(e^{j\omega}) = e^{j\omega \omega_0} \text{rect} \left( \frac{\omega}{2\omega_c} \right) \]

DT domain

\[ h_{LP}[n] = \frac{\sin \omega_c (n-n_0)}{\pi (n-n_0)} \quad -\infty < n < \infty \]

Let \( n_0 = N/2 \)

\[ \hat{h}_{LP}[n] = \frac{\sin \omega_c (n-N/2)}{\pi (n-N/2)} \quad \text{for} \quad 0 \leq n \leq N \]

\[ H_{LP}(e^{j\omega}) = \sum_{n=0}^{N} \hat{h}_{LP}[n] e^{j\omega n} = e^{-j\omega N/2} H_{LP}(\omega) \]
Frequency sampling (FS)

1. Specify desired frequency
2. Sample
3. Take inverse to get \( h[n] \)

Given a desired transfer \( H_d(\omega) \) sample at \( \omega_k = \frac{2\pi}{M} (k+x) \) for \( k = 0, 1, \ldots, \frac{M-1}{2} \), \( M \) odd

or \( k = 0, 1, \ldots, \frac{M-1}{2} \), \( M \) even

let \( x = 0 \) or \( \frac{1}{2} \)

**NOTE:** There are generally 4 variations dependent on symmetry and odd or even length