Consider a shifted delta

DFT of $6\delta(n-p)$

Kronecker delta

Discrete Fourier Transform (DFT)

Continuous Time Fourier Transform (CTFT)

$$\text{CTFT: } \mathcal{X}(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} \, dt$$

$$\text{DFT: } X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi nk/N}$$

Discrete Fourier Transform (DFT)

$$a[n] = \frac{1}{N} \sum_{k=0}^{N-1} \frac{e^{j2\pi nk}}{\sqrt{N}}$$
Discrete Time Cosine and sine wave

\[ X_I(n) = \cos \frac{2\pi kn}{N} \]
\[ X_Q(n) = \sin \frac{2\pi kn}{N} \]

\[ X_I[k] = \sum_{n=0}^{N-1} \cos \left( \frac{2\pi kn}{N} \right) e^{-j \frac{2\pi kn}{N}} \]

\[ = \frac{1}{2} \sum_{n=0}^{N-1} e^{-j \frac{2\pi kn}{N}} e^{-j \frac{2\pi k}{N}} + \frac{1}{2} \sum_{n=0}^{N-1} e^{-j \frac{2\pi kn}{N}} e^{j \frac{2\pi k}{N}} \]

\[ = \frac{1}{2} \sum_{n=0}^{N-1} \delta [k - k_c] + \delta [k + k_c] \]

\[ \frac{1}{N - k_c} \]

**CTFT**

\[ \mathcal{F} \left\{ \cos(2\pi ft_c) \right\} = \frac{\delta (f - f_c) + \delta (f + f_c)}{2} \]

\[ \frac{d}{dt} \cos(2\pi ft_c) = -2\pi f_c \sin(2\pi ft_c) \]
\[ h[n] = \begin{cases} 1 & n \neq 0, \text{ else otherwise} \\ 0 & n = 0 \end{cases} \]

\[ H[k] = \sum_{n=-\infty}^{\infty} h[n] e^{-j2\pi nk} \]

\[ H[k] = \sum_{n=-\infty}^{\infty} \delta[n-k] e^{-j2\pi nk} \]

\[ = \sum_{n=0}^{\infty} e^{-j2\pi nk} - \sum_{n=\infty}^{-1} e^{-j2\pi nk} + \sum_{n=-1}^{0} e^{-j2\pi nk} \]

\[ = 2 \delta[k] \quad k \neq 0 \]

\[ H[k] = \begin{cases} 2 & k = 0 \\ 0 & k \neq 0 \end{cases} \]

\[ DFT: \quad H[k] = \sum_{n=0}^{N-1} h[n] e^{-j2\pi nk/N} \]

\[ DFT: \quad H[k] = \sum_{n=0}^{N-1} \delta[n-k] e^{-j2\pi nk/N} \]

\[ = 2 \sin(\pi k/N) \quad \frac{N-1}{2} \]

\[ DFT: \quad H[k] = \sum_{n=0}^{N-1} \delta[n-k] e^{-j2\pi nk/N} \]

\[ = 2 \sin(\pi k/N) \quad \frac{N-1}{2} \]