EE630: Lecture 25

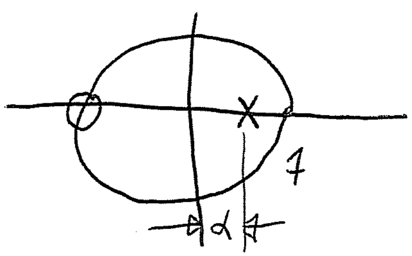
Infinite Impulse Response (IIR) filters

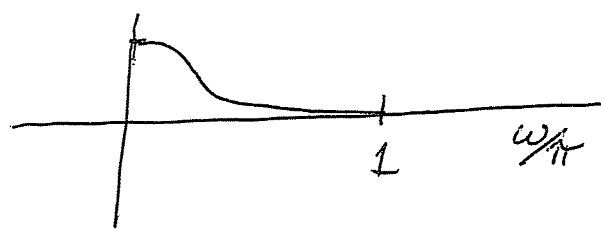
First order

$$H_{LP}(z) = \frac{1-\alpha}{2}\,\frac{1+z^{-1}}{1-\alpha z^{-1}} \quad \text{for } |\alpha| < 1$$

<u>Stability Condition</u>

$$S = \sum_n |h[n]| < \infty$$





$$|H_{LP}(e^{j\omega})|^2 = \frac{(1-\alpha)^2(1+\cos\omega)}{2(1+\alpha^2-2\alpha\cos\omega)}$$

½ power cutoff

$$\tfrac{1}{2} = |H_{LP}(e^{j\omega})|^2 \Rightarrow (1-\alpha)^2(1+\cos\omega_c) = 1+\alpha^2-2\alpha\cos\omega_c$$

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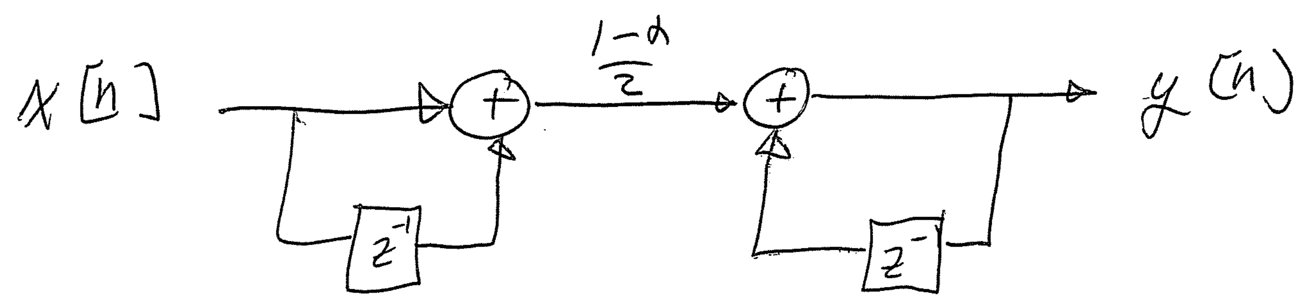
$$\cos w_c = \frac{2\alpha}{1+\alpha^2} \Rightarrow \text{stable solution} \quad \alpha = \frac{1-\sin\omega_c}{\cos\omega_c}$$

We obtain a difference equation as:

$$Y(z)\,2\,(1-\alpha z^{-1}) = X(z)\quad(1-\alpha)(1+z^{-1})$$

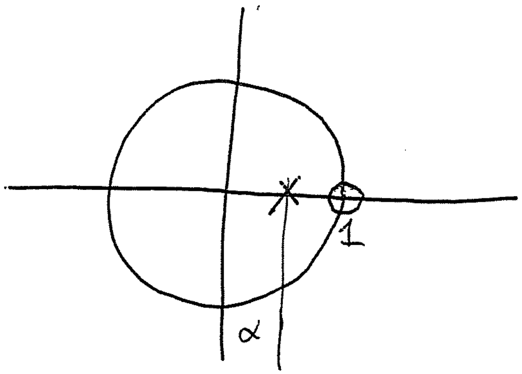
$$Y(z) = \frac{\alpha z^{-1}}{2} Y(z) + \left(\frac{1-\alpha}{2}\right)\left(X(z) + z^{-1}X(z)\right)$$

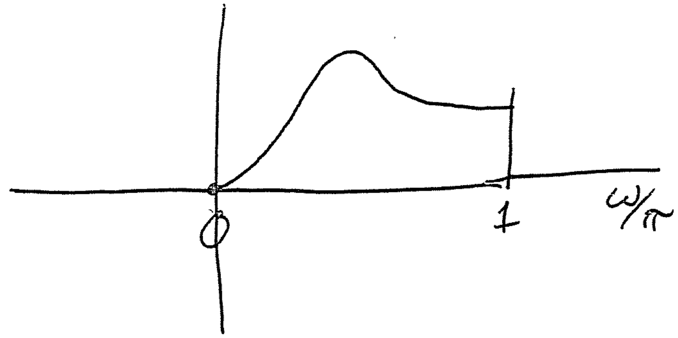
$$y[n] = \alpha/_2\, y[n-1] + \frac{1-\alpha}{2} x[n] + \frac{1-\alpha}{2} x[n-1]$$



High pass IIR Filter

$$H_{HP}(z) = \frac{1+\alpha}{2}\,\frac{(1-z^{-1})}{(1-\alpha z^{-1})} \qquad |\alpha|<1$$



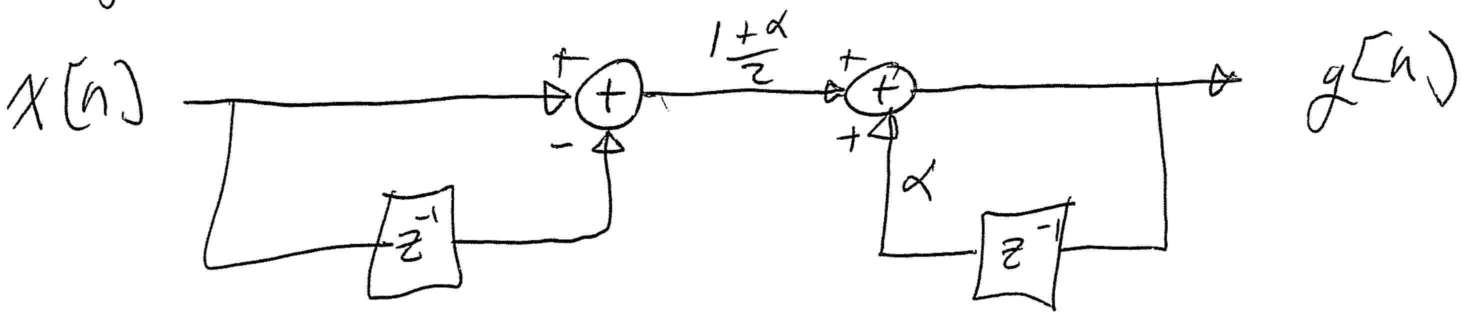


Find the difference equation

$$\left(1 - \alpha z^{-1}\right) Y(z) = \frac{1+\alpha}{2}\left[X(z) - z^{-1} X(z)\right]$$

$$Y(z) = \alpha z^{-1} Y(z) + \frac{1+\alpha}{2}\left[\quad '' \qquad '' \quad\right]$$

$$y(n) = \alpha\, y[n-1] + \frac{1+\alpha}{2}\left(x[n] - x[n-1]\right)$$

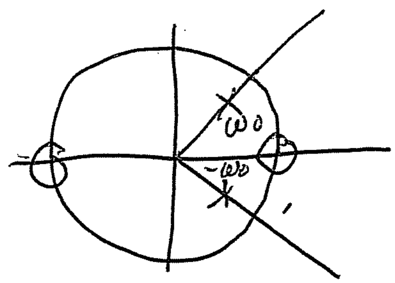


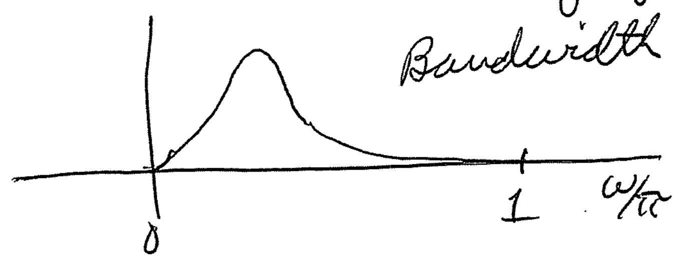
Band pass IIR (second order)

$$H_{BP}(z) = \frac{1-\alpha}{2}\,\frac{1 - z^{-2}}{1 - \beta(1+\alpha)z^{-1} + \alpha z^{-2}} \qquad |\alpha| < 1$$

$$|H_{BP}(e^{j\omega})|^2 = \frac{(1-\alpha)^2(1-\cos 2\omega)}{2\left[1 + \beta^2(1+\alpha)^2 + \alpha^2 - 2\beta(1+\alpha)^2 \cos\omega + 2\alpha\cos 2\omega\right]}$$

zeros @ $\omega = 0, \omega = \pi$





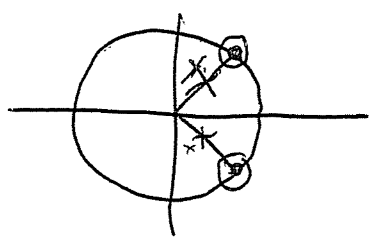
center frequency I $\omega_0 = \cos^{-1}(\beta)$

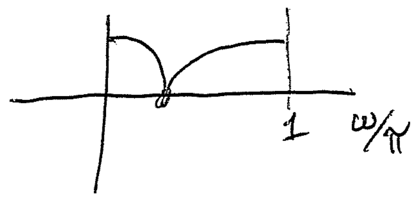
Bandwidth cutoff $B_\omega = \omega_{c2} - \omega_{c1}$

$= \cos^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right)$

Band stop IIR

$$H_{BS}(z) = \frac{1+\alpha}{2}\,\frac{1 - 2\beta z^{-1} + z^{-2}}{1 - \beta(1+\alpha)z^{-1} + \alpha z^{-2}}$$





High order IIR

$$G_{LP}(z) = \left(\frac{1-\alpha}{2} \frac{1+z^{-1}}{1-\alpha z^{-k}} \right)^K$$

Half power freq

at $$\alpha = \frac{1 + (1-C)\cot \omega_c - \sin \omega_c \sqrt{2C-C^2}}{1 - C + \cot \omega_c}$$

where $C = 2^{\left(\frac{K-1}{K}\right)}$

Comb filter

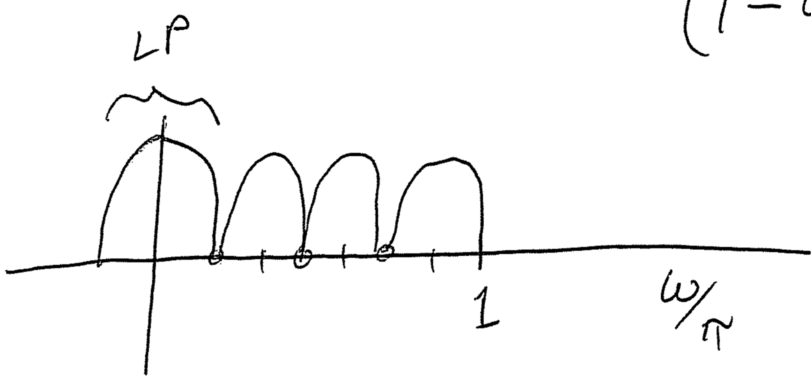
Recall FIR Comb filters $$H(z) = \underbrace{\frac{1-z^{-M}}{M(1-z^{-1})}}_{\text{moving average}}$$

comb filter is $$H(z^L) = \frac{1-z^{-LM}}{M(1-z^{-L})}$$

IIR Comb

(HPF) $H_0(z) = K \dfrac{1-z^{-1}}{1-\alpha z^{-1}}$

Comb $H_0(z^L) = \dfrac{K(1-z^{-L})}{(1-\alpha z^{-L})}$



(LPF) $H_1(z) = \dfrac{1+z^{-1}}{1-\alpha z^{-1}}$

$H_1(z^L) = K \dfrac{1+z^{-L}}{1-\alpha z^{-L}}$