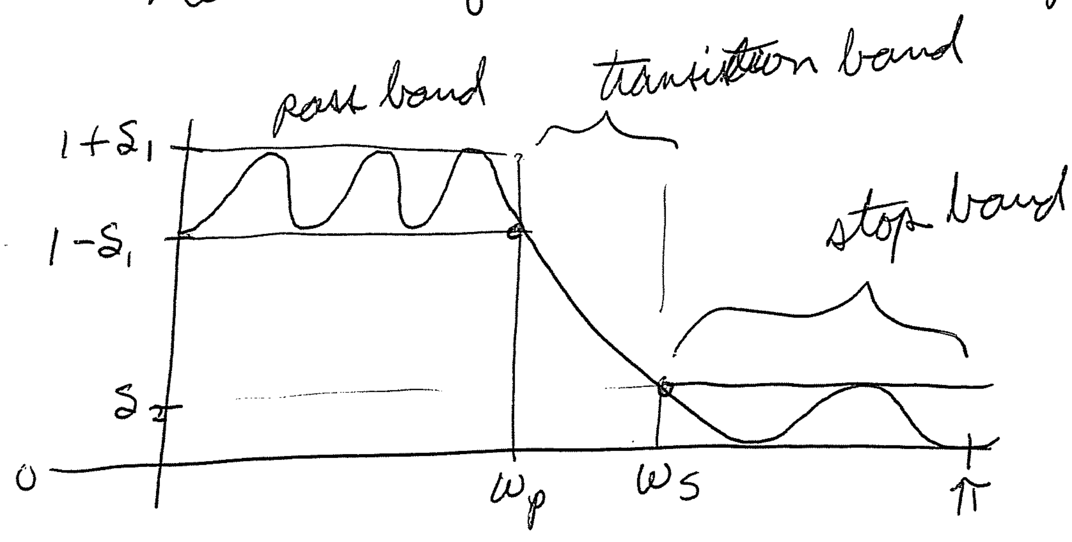
EE630 : Lecture 23 FIR Design ①-09

Least squared time sample, frequency Sample



Design of FIR filter

$$y[n] = \sum_{k=0}^{M-1} b_k x[n-k] \quad \text{or} \quad y[n] = \sum_{k=0}^{M-1} h[k]x[n-k]$$

$$H(z) = \sum_{k=0}^{M-1} h[k] z^{-n}$$

An FIR has linear phase if it is

Symmetric $h[n] = h[M-1-n]$

or

Anti-Symmetric $h[n] = -h[M-1-n]$

In the z-domain these properties are

$$z^{-(M-1)} H(z^{-1}) = \pm H(z)$$

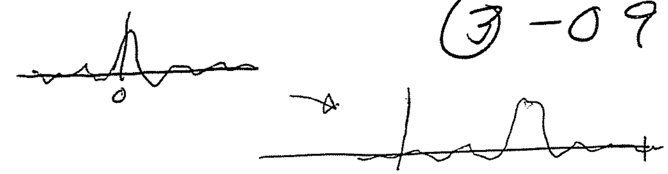
The above implies that the roots of $H(z)$ are equal to the roots of $H(z^{-1})$

i.e., z_1 is a zero of $H(z)$ then $\frac{1}{z_1}$ is a zero of $H(z^{-1})$

Least Square design (time sample)

Ex: Desired frequency response is

$$H_{LP}(e^{j\omega}) = e^{j\omega n_0} \, \mathrm{rect}\left(\frac{\omega}{2\omega_c}\right)$$



In DT domain

$$h_{LP}[n] = \frac{\sin \omega_c (n-n_0)}{\pi (n-n_0)}, \quad -\infty < n < \infty$$

Let $n_0 = N/2$ and shift to center in selection window

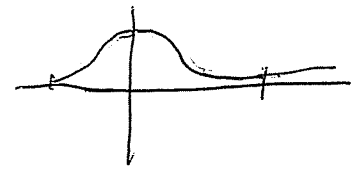
$$\hat{h}_{LP}[n] = \frac{\sin \omega_c (n - N/2)}{\pi (n - N/2)} \quad \text{for } 0 \le n \le N$$

$$H_{LP}(e^{j\omega}) = \sum_{n=0}^{N} \hat{h}_{LP}[n] e^{-j\omega n} = e^{-j\omega N/2} \hat{H}_{LP}(e^{j\omega})$$

$$= e^{-j\omega N/2} \hat{H}_{LP}(e^{j\omega})$$

Frequency Sampling

1. Specify desired frequency
2. Sample the frequency
3. Take inverse to get $h[n]$

Given a desired transfer $H_d(\omega)$ 

sample at $\omega_k = \frac{2\pi}{M}(k+\alpha)$

for $k = 0, 1, \cdots \frac{M-1}{2}$ M odd

or $k = 0, 1 \cdots \frac{M}{2} - 1$ M even

Let $\alpha = 0$ or $1/2$

NOTE: There are generally 4 variations dependent on symmetry and odd or even length