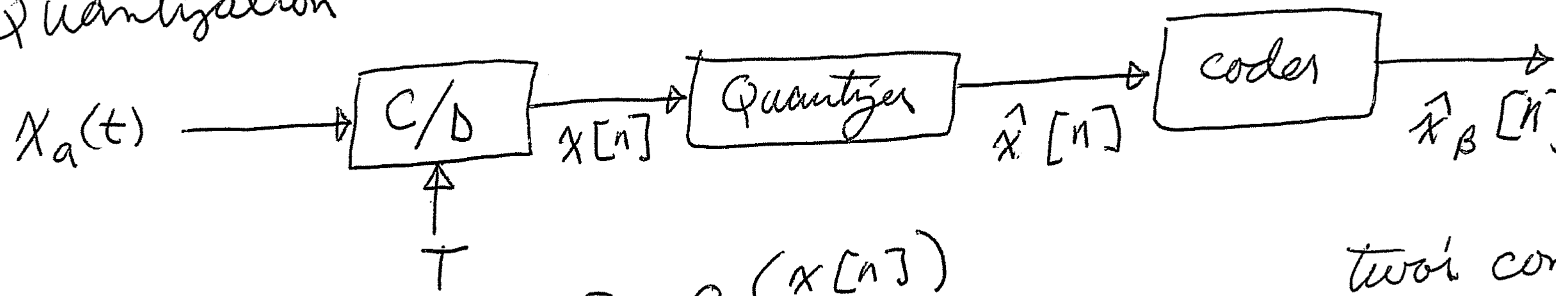
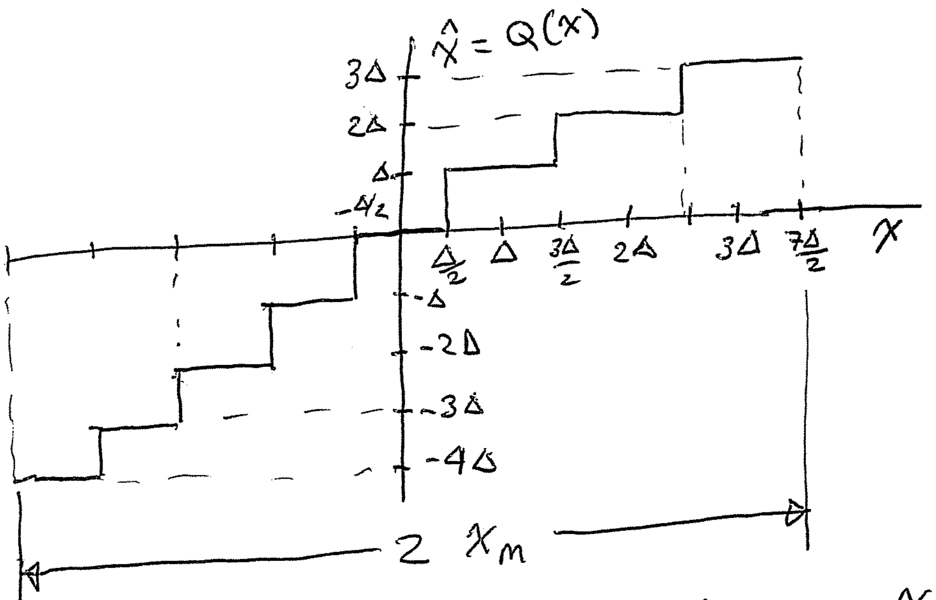
EE 630: Lecture 14 Quantization Model

Quantization



$\hat{x}[n] = Q(x[n])$



two's compliment $\hat{x}$

a_0	a_1	a_2
0	1	1
0	1	0
0	0	1
0	0	0
1	1	1
1	1	0
1	0	1
1	0	0

Full scale: $\Delta = \frac{2X_m}{2^{B+1}} = \frac{X_m}{2^B}$

so if $B = 2$ bits $\Delta = \frac{X_m}{2^2} = \frac{X_m}{4}$

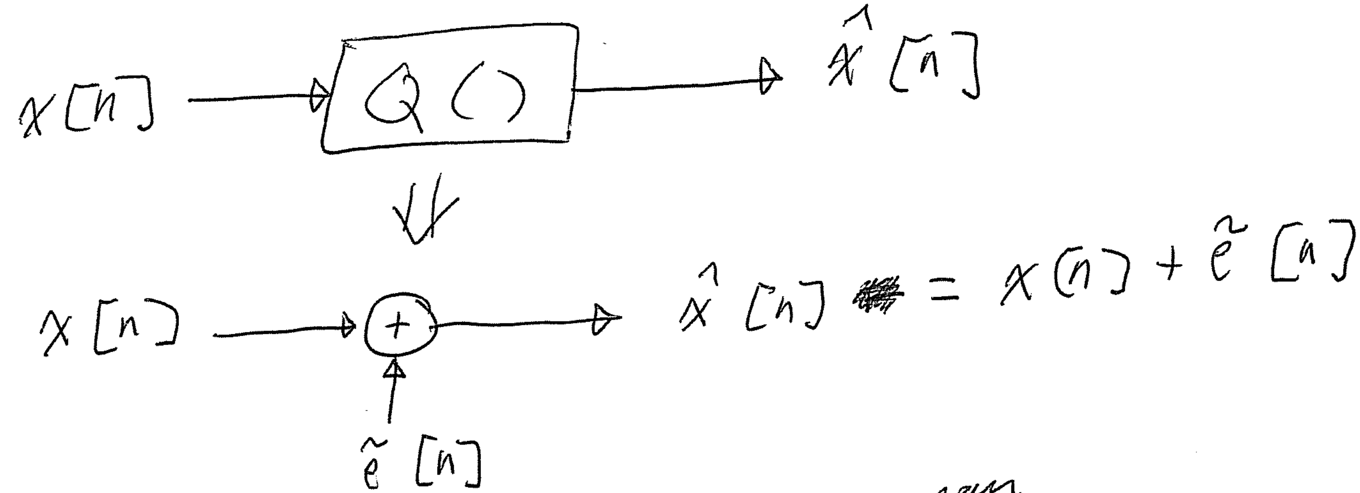
$\hat{x}[n] = X_m \hat{x}_B[n]$

Analysis

Let Q error be $e[n] = \hat{x}[n] - x[n]$

where we assume $x(n)$ is sufficiently complicated to let $e[n]$ ~~be~~ approximate a noise sequence $\tilde{e}[n]$

Then $\hat{x}[n] = x[n] + \tilde{e}[n]$ such that we have and additive noise model.



We assume

1. $\tilde{e}(n)$ is stationary
2. $E\{\tilde{e}[n]\,\hat{x}[n]\} = 0 = E\{\tilde{e}[n]\}\, x[n]$ (0 mean)
3. $E\{\tilde{e}[n]\,\tilde{e}[m]\}\big|_{n\neq m} = E\{\tilde{e}[n]\}\, E\{\tilde{e}[m]\} = 0$

4. $n = m \quad E\{\tilde{e}^2[n]\} = \sigma_e^2 + \mu_e^2 = \sigma_e^2$

5. $f_e(e) \sim \frac{1}{\Delta} U(-\Delta/2, \Delta/2)$

$$E\{\tilde{e}^2[n]\} = \int_{-\infty}^{\infty} e^2 f_e(e)\,de = \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} e^2\,de = \frac{1}{\Delta}\frac{e^3}{3}\Big|_{-\Delta/2}^{\Delta/2}$$

$$= \Delta^2/12 = \sigma_e^2$$

The SNR is $\quad SNR = \dfrac{\sigma_x^2}{\sigma_e^2}$

and $\Delta = \dfrac{X_m}{2^B}$, $\boxed{\sigma_e^2 = \dfrac{X_m^2\, 2^{-2B}}{12}}$

$$SNR = \frac{\sigma_x^2\, 2^{2B}\, 12}{X_m^2} = 10\log_{10}\frac{\sigma_x^2\, 2^{2B}\, 12}{X_m^2} = 6.02B + 10.8 - 20\log_{10}\Big(\underbrace{\frac{X_m}{\sigma_x}}_{\text{full range}}\Big)$$

where σ_x is rms value of signal amplitude

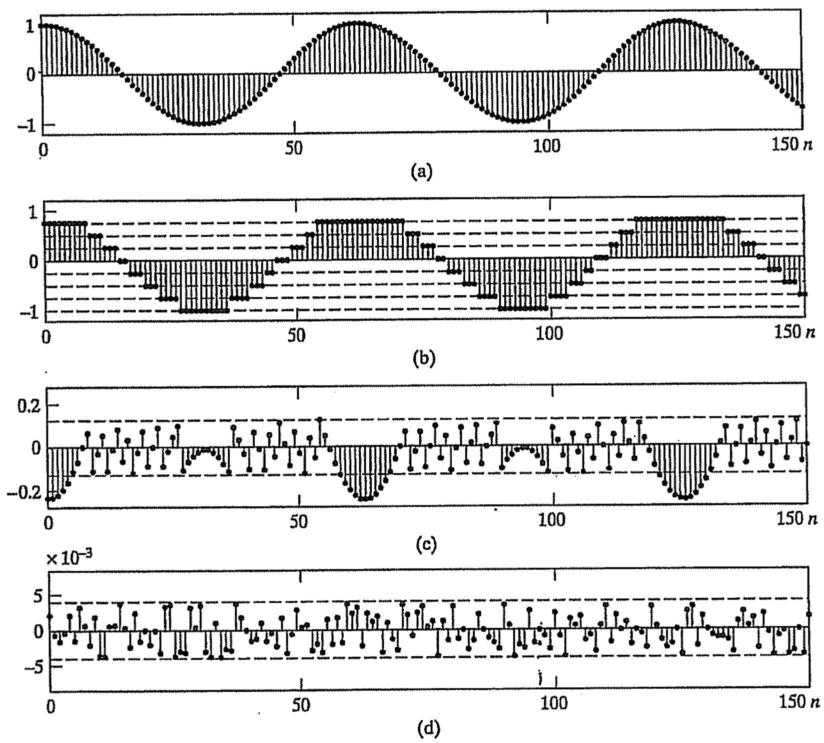


Figure 4.51 Example of quantization noise. (a) Unquantized samples of the signal $x[n] = 0.99\cos(n/10)$. (b) Quantized samples of the cosine waveform in part (a) with a 3-bit quantizer. (c) Quantization error sequence for 3-bit quantization of the signal in (a). (d) Quantization error sequence for 8-bit quantization of the signal in (a).

and they range between $-\Delta/2$ and $+\Delta/2$. In Chapter 10 this will be demonstrated more quantitatively when we calculate the power density spectrum and autocorrelation of a quantization-noise sequence.

For quantizers that round the sample value to the nearest quantization level, as shown in Figure 4.48, the amplitude of the quantization noise is in the range

$$-\Delta/2 < e[n] \leq \Delta/2. \tag{4.121}$$

For small Δ, it is reasonable to assume that $e[n]$ is a random variable uniformly distributed from $-\Delta/2$ to $\Delta/2$. Therefore, the first-order probability density for the quantization noise is as shown in Figure 4.52. (If truncation rather than rounding is used in implementing quantization, then the error would always be negative, and we would assume a uniform probability density from $-\Delta$ to 0.) To complete the statistical model for the quantization noise, we assume that successive noise samples are uncorrelated with each other and that $e[n]$ is uncorrelated with $x[n]$. Thus, $e[n]$ is assumed to be a uniformly distributed white-noise sequence. The mean value of $e[n]$ is zero, and its