EE630: Lecture 12 DTFT properties ①-09

Linearity: $a_1 x_1[n] + a_2 x_2[n] \xleftrightarrow{DTFT} a_1 X(\omega) + a_2 X(\omega)$

Time shifting: $x[n-k] \longleftrightarrow e^{-j\omega k} X(\omega)$

Time reversal: $x[-n] \longleftrightarrow X(-\omega)$

? does $x[-n]$ need to be real

Convolution

$x[n] = x_1[n] * x_2[n] \longleftrightarrow X(\omega) = X_1(\omega) X_2(\omega)$

$= \sum_k x_1[k] x_2[n-k]$

Correlation

$r_{x_1 x_2}[m] \longleftrightarrow S_{x_1 x_2}(\omega) = X_1(\omega) X_2(-\omega)$

does $x[+n]$ is real?

$r_{x_1 x_2}[n] = \sum_k x_1[k] x_2[k-n]$

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Wiener - Khintchine theorem

$$r_{xx}(\ell) \longleftrightarrow S_{xx}(\omega)$$

Frequency shifting

$$e^{j\omega_0 n} x[n] \longleftrightarrow X(\omega - \omega_0)$$

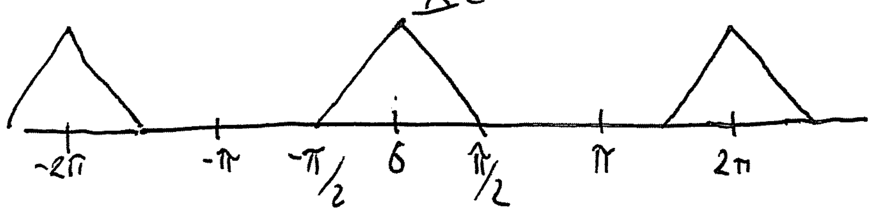
Modulation

$$x[n] \cos \omega_0 n \longleftrightarrow \frac{1}{2}\left(X(\omega + \omega_0) + X(\omega - \omega_0) \right)$$

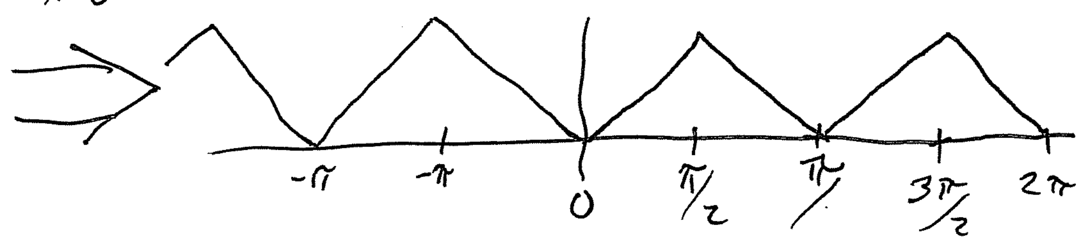
proof: $\cos \omega_0 n = \frac{1}{2}\left(e^{j\omega_0 n} + e^{-j\omega_0 n} \right)$

so $$\frac{x[n] e^{j\omega_0 n}}{2} + \frac{x[n] e^{-j\omega_0 n}}{2} \longleftrightarrow \frac{X(\omega - \omega_0) + X(\omega + \omega_0)}{2}$$

$X(\omega)$



$\times \cos \omega_0 n$ in DT where $\cos 0.5\pi n$



Parseval's theorem

$$\sum_n x_1[n]\, x_2^*[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(\omega)\, X_2^*(\omega)\, d\omega$$

If $x_1[n] = x_2[n] = x[n]$

$$\sum_n |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(\omega)|^2\, d\omega$$

Multiplication of two sequences (Windowing theorem)

$$x_3[n] \equiv x_1[n]\, x_2[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(\lambda)\, X_2(\omega-\lambda)\, d\lambda$$

Differentiation in freq. domain

$$n\,x[n] \longleftrightarrow j\,\frac{dX(\omega)}{d\omega}$$

see Table 4.5 at page 304