We will discuss and define three types of sampling models:

1. Impulse sampling
2. Natural Sampling
3. Flat Top sampling

Impulse sampling

Consider a CT signal \( g(t) \)

We impulse sample \( g(t) \), by multiplying \( g(t) \) by a train of dirac delta functions \( t \in T \).

\[
q_s(t) = g(t) \sum_{n} \delta(t-nT)
\]

where \( T \) is the sample period.
The DT values $g[n]$ are the non-zero values of $g_s(t)$, i.e., $g[n] = g_s(nT)$.

The block diagram is $g(t) \rightarrow g_s(t) \rightarrow \text{CT & DT} \rightarrow g[n]$. 

Ex:

$$p(t) = \sum_{n} \delta(t-nT)$$

$$g_s(t)$$
To convert $g[n]$ back to CT, another impulse train is multiplied s.t.

$$\hat{g}_s(t) = \sum_{n} g[n] \delta(t-nT)$$

The reconstructed signal $\hat{g}(t)$ is obtained by

$$g[n] \rightarrow \hat{g}_s(t) \rightarrow \text{LPF} \quad \rightarrow \hat{g}(t)$$

The frequency domain reveals the limitations and capabilities of impulse sampling.

Assume $G_1(f)$ is bandlimited s.t.

$$|G_0(f)|$$

then $G_s(f) = G_1(f) \ast \frac{1}{T} \leq \delta(f - \frac{k}{T})$

$$\Rightarrow G_s(f) = \frac{1}{T} \leq \delta(f - \frac{k}{T})$$
Ex: Allowing

Aliasing occurs \( @ \) \( B = \frac{1}{2} - B \)

so no aliasing

\( t < \frac{1}{2}B \) or \( f_5 = \frac{1}{T} > 2B \)
If there is NO aliasing we can reconstruct using an ideal low-pass filter

\[ \frac{1}{T} |G(\omega)| \]

\[ f \]

\[ T H_1(f) \]

\[ |G(\omega)| \]

\[ f \]

Note that:

\[ g_s(t) = g(t) \sum_h s(t-hT) \]

\[ = \sum_h g(\eta T) s(t-hT) \]

\[ = \sum_h g[h] s(t-hT) \]
Also note: The DTFT of \( g[n] \) is

\[
G(e^{j\omega}) = \sum_{n} g[n] e^{-j\omega n} \Rightarrow \text{Note } G(z) = \sum_{n} g[n] z^{-n} \text{ if } z = e^{j\omega},
\]

then \( G(e^{j\omega}) = \sum_{n} g[n] e^{-j\omega n} \).

\[
\text{which is proportional to } G_{5}(f) \text{ for } \frac{2\pi}{T} = 2\pi \text{ or } T = 1.
\]
Anti-Aliasing

We can prevent aliasing using anti-aliasing filter at the cost of losing bandwidth $t_i$. 

\[ g(t) \xrightarrow{} H_a(f) \xrightarrow{} g_a(t) \xrightarrow{} g_{as}(t) \]

\[ p(t) \]

where

\[ G(f) \]

\[ -B \quad 0 \quad B \]

\[ H_a(f) \]

\[ -\frac{1}{2T} \quad \frac{1}{2T} = B_H \]

\[ G_{as}(f) \]

\[ -\frac{1}{4} \quad 0 \quad \frac{1}{4} \]