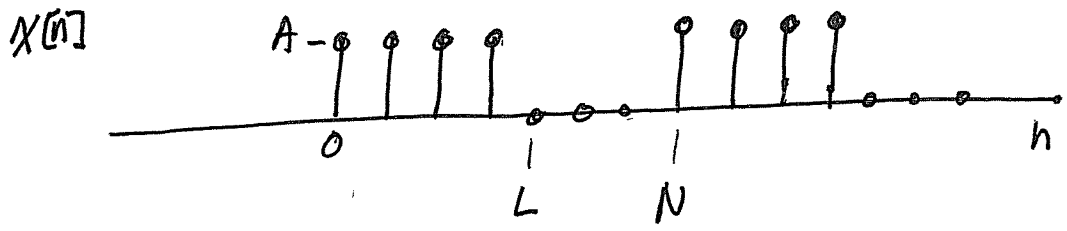
EE630
Lecture 11B: DTFT of square wave

①-09



In the text we have what is called the DT Fourier Series or DTFS as

Synthesis for periodic waves $$x[n] = \sum_{k=0}^{N-1} C_k e^{j\frac{2\pi k n}{N}}$$

Analysis equation $$C_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi k n}{N}}$$

where $k = 0, 1 \ldots (N-1)$

Using $x(n)$ as a square wave

$$C_k = \frac{1}{N} \sum_{n=0}^{L-1} A e^{-j 2\pi k n / N}$$

we know the geometric series is $$\sum_{n=0}^{N-1} a^n = \begin{cases} N & a = 1 \\ \frac{1-a^N}{1-a} & a \neq 1 \end{cases}$$

so
$$c_k = \frac{A}{N} \sum_{n=0}^{L-1} \left(e^{-j\frac{2\pi k}{N}}\right)^n = \begin{cases} \dfrac{AL}{N} & k = 0 \\ \dfrac{A}{N}\,\dfrac{1-e^{-j\frac{2\pi k L}{N}}}{1-e^{-j\frac{2\pi k}{N}}} & \text{for } k = 1, 2 \ldots (N-1) \end{cases}$$

Using algebra and Euler's identity we have

$$c_k = \begin{cases} \dfrac{AL}{N} & k = 0, \pm N, \pm 2N \ldots \\ \dfrac{A}{N}\, e^{-j\frac{2\pi k (L-1)}{N}} \dfrac{\sin(\pi k L/N)}{\sin(\pi k/N)} & \text{elsewhere} \end{cases}$$

$$|c_k|^2 = \begin{cases} \left(\dfrac{AL}{N}\right)^2 & \text{———} \\ \left(\dfrac{A}{N}\right)^2 \left(\dfrac{\sin(\pi k L/N)}{\sin(\pi k/N)}\right)^2 & \text{———} \end{cases}$$

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