Lecture 10: EE630

Discrete Time Fourier Transform (DTFT)

Let a sampled signal be

$$x_s(t) = x(t) \sum_n \delta(t - nT)$$

$$\mathcal{F}\{x_s(t)\} = \mathcal{F}\left\{x(t) \sum_n \delta(t-nT)\right\}$$

$$= \int_{-\infty}^{\infty} x(t) \sum_n \delta(t-nT) e^{-j2\pi f t} dt$$

$$= \sum_n \int_{-\infty}^{\infty} x(t)\delta(t-nT) e^{-j2\pi f t} dt$$

$$= \sum_n x(nT) e^{-j2\pi f n T}$$

let $x[n] = x(nT)$

To simplify we let $T = 1$ sec

so $\mathcal{F}_t\{x_s(t)\} = \sum_n x[n]\, e^{-j\omega n} = \text{DTFT} = X(\omega)$

where $\omega = 2\pi f$

The DTFT has an inverse

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(\omega)\, e^{j\omega n}\, d\omega$$

The DTFT is a direct outcome of the Z-form ~~evalutet~~ evaluated on the unit circle.

that is $X(z)\Big|_{z=e^{j\omega}} = X(e^{j\omega}) = X(\omega) = \sum_n x[n]\, e^{-j\omega n}$