EE636 Lecture 9

Z-domain Sinsoidal Oscillator

$$H(z) = \frac{b_0}{1 + a_1 z^{-1} + a_2 z^{-2}} = \frac{Y(z)}{X(z)}$$

Let $a_1 = -2r\cos\omega_0$, $a_2 = r^2$

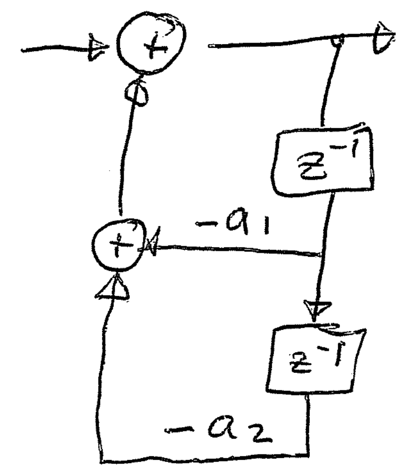
$$p_{oles} = r e^{\pm j\omega_0}$$

$$h[n] = \frac{b_0 r^n}{\sin\omega_0} \left(\sin (n+1)\omega_0\right) u[n]$$

if $r = 1$ and $b_0 = \sin\omega_0$ then

$$h[n] = A \sin (n+1)\omega_0 \; u[n]$$

$x[n] = (A \sin \omega_0)\,\delta[n]$



$y[n] = A \sin (n+1)\,\omega_0$

assume $y[n]=0$ for $n<0$

$$Y(z)\left(1 + a_1 z^{-1} + a_2 z^{-2}\right) = b_0 X(z)$$

$$y[n] = b_0\, x[n] - a_1\, y[n-1] - a_2\, y[n-2]$$

let $y[0] = A \sin \omega_0$

$$y[1] = (2 \cos \omega_0)\, y[0] = 2 \cos\omega_0\, \cancel{A} \sin \omega_0 = A \sin 2\omega_0$$

$$y[2] = (2 \cos \omega_0)\, y[1] - y[0] = A \sin 3\omega_0$$

In some applications we need both cosine and sine waveforms for ~~quadrature~~ quadrature processing.

Coupled form generates a synchronized quadrature pair.

Note:
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

Let $\alpha = n\omega_0$, $\beta = \omega_0$

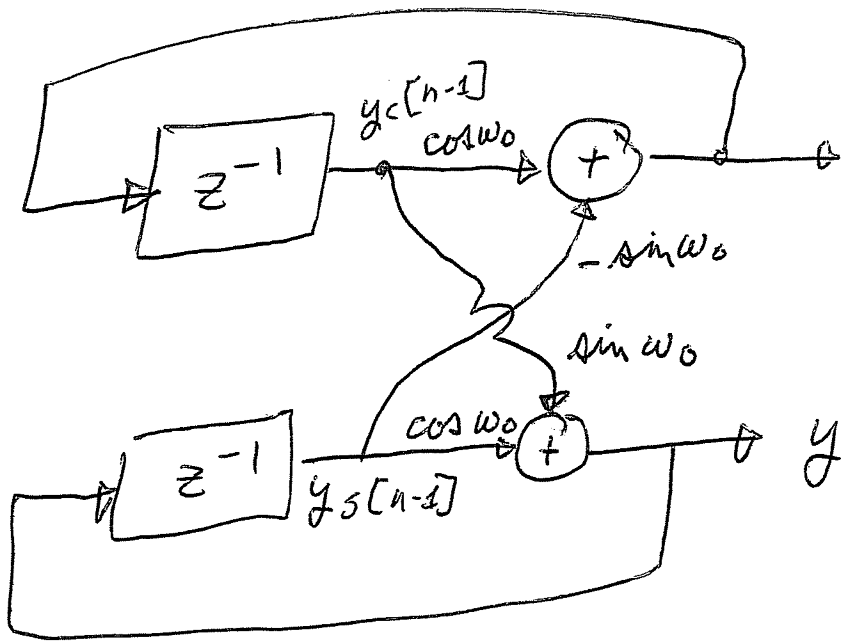
$$y_c[n] = \cos(n\omega_0)\, u[n]$$
$$y_s[n] = \sin(n\omega_0)\, u[n]$$

$$\begin{cases} n\omega_0 = (n-1)\omega_0 + \omega_0 \\ y_c[n-1] = \cos((n-1)\omega_0) \\ y_s[n-1] = \sin((n-1)\omega_0) \end{cases}$$

$$y_c[n] = \cos\omega_0\, y_c[n-1] - (\sin\omega_0)\, y_s[n-1]$$
$$y_s[n] = \sin\omega_0\, y_c[n-1] + (\cos\omega_0)\, y_s[n-1]$$

or

$$\begin{bmatrix} y_c[n] \\ y_s[n] \end{bmatrix} = \begin{bmatrix} \cos \omega_0 & -\sin \omega_0 \\ \sin \omega_0 & \cos \omega_0 \end{bmatrix} \begin{bmatrix} y_c[n-1] \\ y_s[n-1] \end{bmatrix}$$



$y_c[n] = \cos \omega_0 n$

$y_s[n] = \sin \omega_0 n$