Lecture 2A: Power and Energy EE511 #1

"Other uses of time integration"

"dc value"

$$W_{dc} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} w(t)\, dt = \lim_{T \to \infty} \langle w(t) \rangle$$

Ex: $\langle \cos 2\pi f_c t \rangle \Big|_{T = 1/f_c} = 0$

<u>Power</u> If we can seperate signal power from noise power sufficiently, we can detect the signal. Thus, we need a definition of power.

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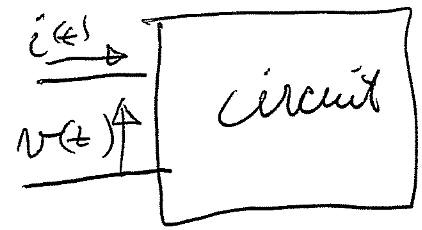
Ex: Consider an electronic device s.t. the instantaneous power is

$$p(t) = \underbrace{v(t)}_{\text{voltage}} \; \underbrace{i(t)}_{\text{current}}$$



average power is

$$P = \langle p(t) \rangle = \frac{1}{T} \int_0^T v(t)\, i(t)\, dt$$

Ex: $v(t) = V \cos 2\pi f_0 t$, $i(t) = I \cos 2\pi f_0 t$

instantaneous power

$$p(t) = VI \cos^2 2\pi f_0 t = VI \left(\frac{1 + \cos 4\pi f_0 t}{2} \right)$$

average power $P = \langle p(t) \rangle = \frac{VI}{2} \frac{1}{T} \int_0^T (1 + \cos 4\pi f_0 t)\, dt$

$$P = \frac{VI}{2}\left(\langle 1\rangle + \underbrace{\langle \cos 4\pi f_0 t\rangle}_{0}\right)$$

$$= \frac{VI}{2}$$

RMS Value

$$W_{rms} = \sqrt{\langle w^2(t)\rangle}$$

given $v(t)$ and $i(t) = \frac{v(t)}{R}$

let $p(t) = v(t)\,i(t) = \frac{v^2(t)}{R} = i^2(t)R$

$$P = \frac{\langle v^2(t)\rangle}{R} = \langle i^2(t)\rangle R = \frac{V_{rms}^2}{R} = I_{rms}^2 R$$

since $\sqrt{\frac{V_{rms}^2}{R}} = \sqrt{I_{rms}^2 R}$

then $\frac{V_{rms}^2}{R} = \sqrt{\frac{V_{rms}^2}{R}} \sqrt{I_{rms}^2 R} = V_{Rms} I_{rms}$

Normalized power : assume $R = 1$

The average normalized power is

$$P = \lim_{T \to \infty} \langle w^2(t) \rangle = \lim_{T \to \infty} \frac{1}{T} \int_0^T \underbrace{w^2(t)}_{v^2(t) \text{ or } i^2(t)} dt$$

Energy

$$E = \lim_{T \to \infty} T \langle w^2(t) \rangle = \lim_{T \to \infty} \int_{-T/2}^{T/2} w^2(t)\, dt$$

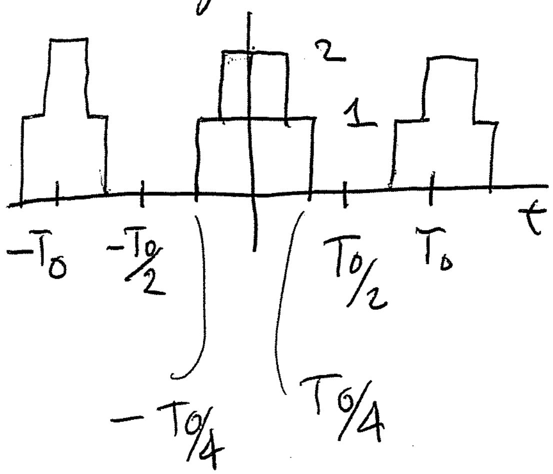
Energy waveform $\quad 0 < E < \infty$

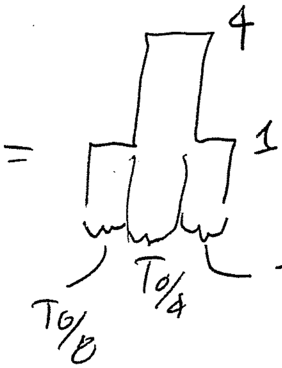
Power waveform $\quad 0 < P < \infty$

Average Power of a periodic signal

$$P = \langle w^2(t) \rangle \Big|_{\text{over period } T} = \frac{1}{T} \int_0^T w^2(t)\,dt$$

Example: $i(t) = \sum_{n=-\infty}^{\infty} \text{rect}\left(\frac{t - nT_0}{T_0/2}\right) + \text{rect}\left(\frac{t - nT_0}{T_0/4}\right)$



$\longrightarrow \quad P_{ave} =$ 

$\frac{1}{T_0} \times \left(4\,\frac{T_0}{4} + 1\,\frac{T_0}{4}\right)$

$= \frac{4}{4} + \frac{1}{4} = \frac{5}{4}$

Ex: Use the rect as a switch (on/off)

$$\text{rect}\left(\frac{t - nT_0}{T_0/2}\right) \sin 2\pi f_c t \quad \text{where } f_c = 4/T_0$$