Phase Modulation (PM)

\[ s(t) = A_c \cos (2\pi f_c t + \phi(t)) = A_c \cos \left( 2\pi \left( f_c t + \frac{\phi(t)}{2\pi} \right) \right) \]

where \( \phi(t) = D_p m(t) \)

where \( D_p \) is the "phase sensitivity"

Frequency Modulation (FM)

same as PM but \( \phi(t) = D_f \int_{-\infty}^{t} m(\tau) d\tau \)

where \( D_f \) is the "frequency deviation constant"

let \( A(t) = f_c t + \frac{\phi(t)}{2\pi} \) then \( \frac{dA(t)}{dt} = f_c \)
We can relate PM and FM by noting

\[ m_f(t) = \frac{Df}{D_t} \frac{d m_f(t)}{dt} \]

or

\[ m_p(t) = \frac{Df}{D_p} \int_{-\infty}^{t} m_f(\tau) \, d\tau \]

These are used as follows:

\[ m_f(t) \rightarrow \text{Integrator} \left\{ \text{gain } \frac{Df}{D_p} \right\} \rightarrow m_p(t) \rightarrow PM \rightarrow \text{FM signal out} \]

\[ m_p(t) \rightarrow \text{Differentiator} \left\{ \text{gain } \frac{Dp}{Df} \right\} \rightarrow m_f(t) \rightarrow \text{FM} \rightarrow \text{PM signal out} \]
Narrow Band FM

Let \( 0(t) \) be small s.t. \(|0(t)| < 0.2 \text{rad} \)

For \( g(t) = A_c e^{j0(t)} \approx A_c (1 + j0(t)) \)

The modulated waveform is

\[
\mathcal{S}(t) = \text{Real} \left\{ g(t) e^{j\omega_c t} \right\}
\]

So

\[
\mathcal{S}(t) = A_c \cos \omega_c t - A_c O(t) \sin \omega_c t
\]

carrier term

sideband

"AM" type signal
Generation of narrow band FM

\[ m(t) \rightarrow \text{Integrator} \rightarrow O(t) \rightarrow \times \rightarrow \pm \rightarrow NBFM \]

\[ f_c \rightarrow -90^\circ \rightarrow A_c \sin \omega_c t \rightarrow Ac \cos \omega_c t \]

Wide band PM and FM spectra

Let \( \varphi(t) = \beta \sin 2\pi f_m t \)

\[ X_c(t) = A \cos \left( 2\pi f_c t + \beta \sin 2\pi f_m t \right) \]

\[ = A \text{real} \left( e^{j 2\pi f_c t} e^{-j \beta \sin 2\pi f_m t} \right) \]
The Fourier series expansion of $e^{j B \sin 2\pi f_m t}$ in terms of $\omega_m = 2\pi f_m$ is

$$\frac{\omega_m}{2\pi} \sum_{nm} e^{j B \sin \omega_m t} e^{-j n \omega_m t} dx$$

$$= \frac{1}{2\pi} \sum_{-\pi}^{\pi} e^{-j (\pi x - B \sin x)} dx$$

Bessel function

so $e^{j B \sin \omega_m t} = \sum \frac{J_n (\beta)}{n} e^{j \pi n \omega_m t}$

so $x_c(t) = A_c \text{ real } \left[ e^{j \omega_c t} \sum \frac{J_n (\beta)}{n} e^{j \pi n \omega_m t} \right]$

The real part is $x_c(t) + x^*_c(t)$
When \( V = \text{grade crossing deviation} \)

\[
R = \frac{\text{headlight cut off meter}}{\text{vehicle width}}
\]

\[
P = \frac{2}{(V+1)}
\]

Which \( W \) is the measure of the 98% of

Bundled:

Common rail:

FM Bundle:

Above:

\[
A_{c}(t) = A_{c} 
\]
For PM

\[ D = \beta_p = D_p A_m \text{ where} \]
\[ m(t) = A_m \cos \omega m t \]

(FM)

\[ D = \beta_f = \frac{D_p A_m}{2 \pi} \]
either case, the modulator output is the unmodulated carrier, which has frequency components only at the carrier frequency.

In computing the spectrum of the modulator output, our starting point was the assumption that

\[ \phi(t) = \beta \sin \omega_m t \]  

We did not specify the modulator type. The assumed \( \phi(t) \) could represent the phase deviation of a PM modulator with \( m(t) = A \sin \omega_m t \) and an index \( \beta = k_p A \).

### Table 3.3

**Values of \( \beta \) for which \( J_n(\beta) = 0 \) for \( 0 \leq \beta \leq 9 \)**

<table>
<thead>
<tr>
<th>( n )</th>
<th>( J_0(\beta) = 0 )</th>
<th>( \beta_{n0} )</th>
<th>( \beta_{n1} )</th>
<th>( \beta_{n2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( J_0(\beta) = 0 )</td>
<td>2.4048</td>
<td>5.5201</td>
<td>8.6537</td>
</tr>
<tr>
<td>1</td>
<td>( J_1(\beta) = 0 )</td>
<td>0.0000</td>
<td>3.8317</td>
<td>7.0156</td>
</tr>
<tr>
<td>2</td>
<td>( J_2(\beta) = 0 )</td>
<td>0.0000</td>
<td>5.1356</td>
<td>8.4172</td>
</tr>
<tr>
<td>4</td>
<td>( J_4(\beta) = 0 )</td>
<td>0.0000</td>
<td>7.5883</td>
<td>—</td>
</tr>
<tr>
<td>6</td>
<td>( J_6(\beta) = 0 )</td>
<td>0.0000</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>
Figure 5–11. Magnitude spectra for FM or PM with sinusoidal modulation for various modulation indexes.