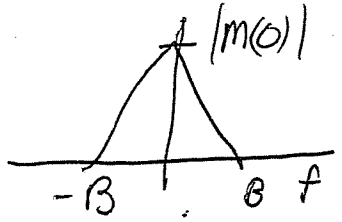
EE511: Lecture 19: Hilbert Transform Vestigial Side Band Modulation

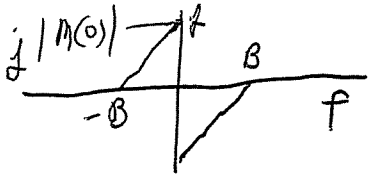
Hilbert Transform: $\hat{m}(t) \triangleq m(t) * h(t)$

where $h(t) = \frac{1}{\pi t} \Rightarrow H(f) = \begin{cases} -j & \text{for } f>0 \\ j & \text{for } f<0 \end{cases} = -j\,\text{sgn}(f)$

let $M(f) =$



then $\hat{M}(f) =$



Let $G_1(f) = A_c\, M(f) \left[1 \pm j\, H(f)\right]$ ← upper or lower side band

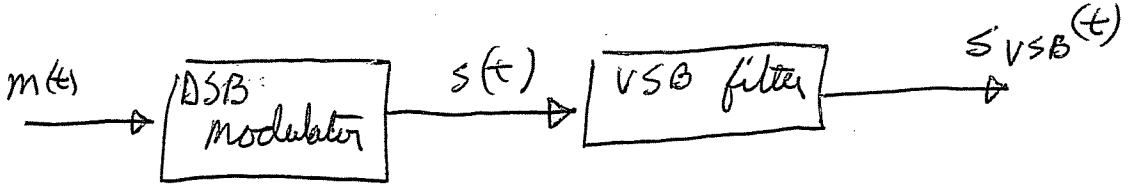
given $V(f) = \frac{1}{2}\left\{G_1(f-f_c) + G_1^*(-(f+f_c))\right\}$

$$S(f) = \frac{A_c}{2}\left[M(f-f_c) + j\,\hat{M}(f-f_c) + M^*(-f-f_c) - j\,\hat{M}(-f-f_c)\right]$$

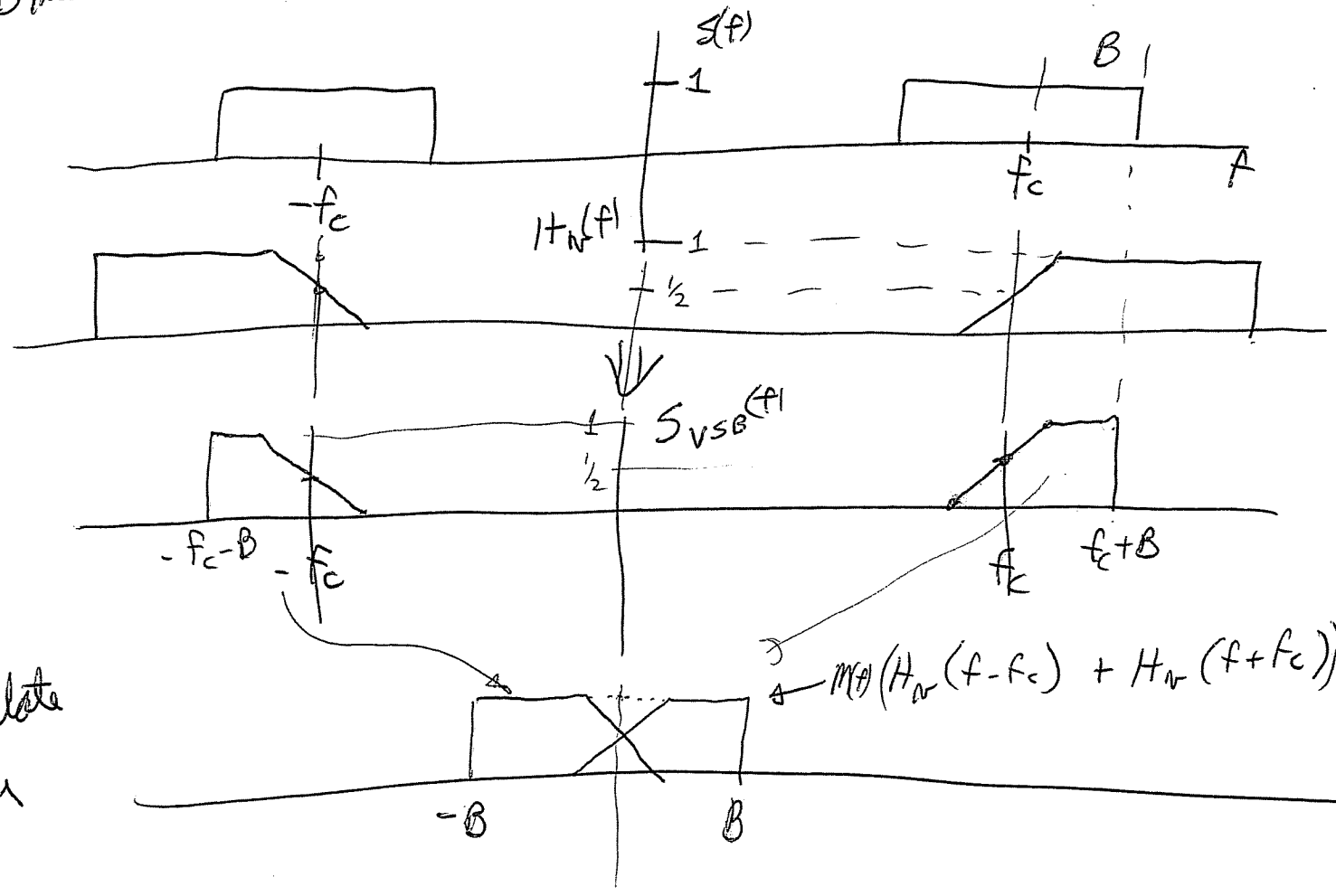
Vestigal Side Band Modulation

Assume we don't have ideal filters to "cut" our lower and upper sidebands apart. We can use the concept of Vestigal Side Band (VSB) in our design. Let $h_V(t)$ be our VSB filter itc

$$s_{VSB}(t) = s(t) * h_V(t) \longleftrightarrow S_{VSB}(f) H_V(f)$$



VSB method


S(f)
1
B
f_c
f
$-f_c$
$|H_V(f)|$
1
½
$S_{VSB}(f)$
1
½
$-f_c-B$
$-f_c$
f_c
f_c+B
$M(f)\left(H_V(f-f_c)+H_V(f+f_c)\right) \stackrel{?}{=} rect\left(\frac{f}{2B}\right)$
demodulate with mixer
$-B$
B