EE511 14C Filter Design

General 2nd order filter expression

$$H(s) = \frac{n_2 s^2 + n_1 s + n_0}{s^2 + (\omega_0/Q)s + \omega_0^2} \qquad (1)$$

$$H(s) = \underbrace{\frac{n_2 s^2}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2}}_{HPF} + \underbrace{\frac{n_1 s}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2}}_{BPF} + \underbrace{\frac{n_0}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2}}_{LPF}$$

n_0, n_1 and n_2 determine zeros in Eq (1) and determine the filter type

The denom: $s^2 + (\omega_0/Q)s + \omega_0^2$ determines poles

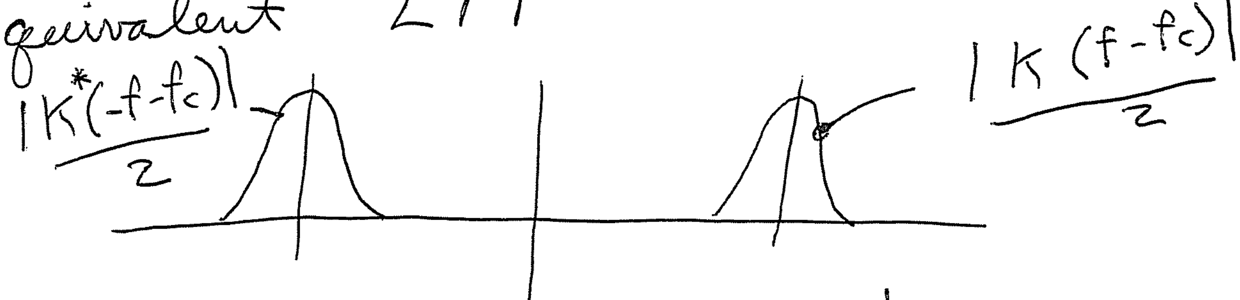
$$p_1, p_2 = -\frac{\omega_0}{2Q} \pm \sqrt{\left(\frac{\omega_0}{2Q}\right)^2 - \omega_0^2}$$

for $Q > 0.5$

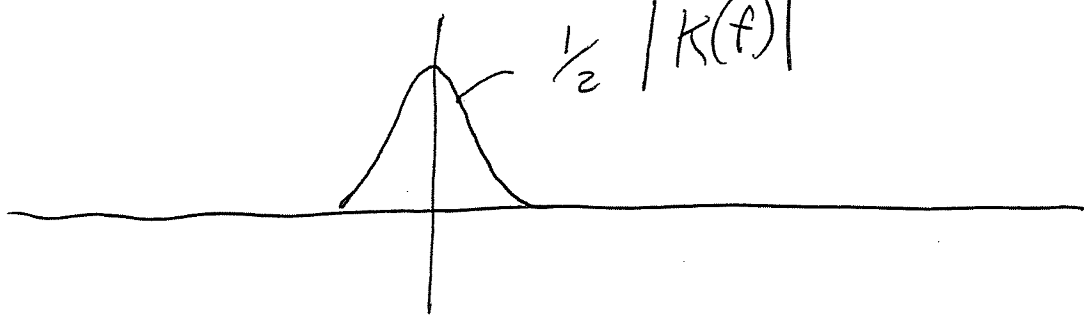
$$p_1, p_2 = -\frac{\omega_0}{2Q} \pm j\sqrt{\omega_0^2 - \left(\frac{\omega_0}{2Q}\right)^2}$$

Equivalent LPF

BP



LP



For EX: given a BPF $H(s) = \dfrac{n_1 s}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2}$

so $H(s) = \dfrac{n_1 s}{(s-p_1)(s-p_2)} = \dfrac{A}{(s-p_1)} + \dfrac{B}{(s-p_2)}$

Use partial fraction expansion (PFE).

Solve for A by mult. by $s-p_1$

$$\frac{n_1 s}{s-p_2} = A + B\frac{(s-p_1)}{(s-p_2)}$$

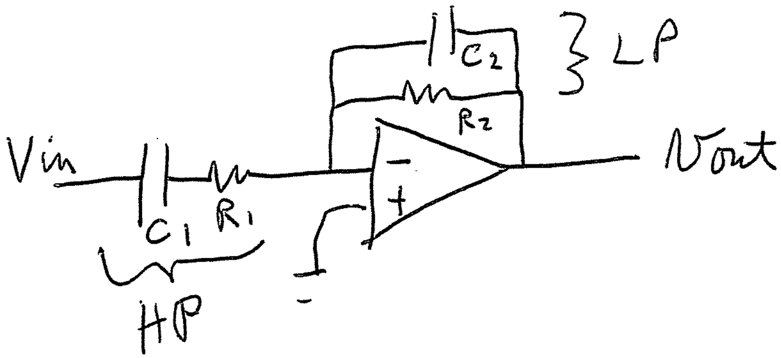
let $s=p_1$ so $A = \dfrac{n_1 p_1}{p_1-p_2}$, likewize $B = \dfrac{n_1 p_2}{p_2-p_1}$

$$H(\omega) = \frac{A}{\underbrace{j(\omega-\omega_0)+C}_{\frac{1}{2}K(\omega-\omega_0)}} + \frac{B}{\underbrace{j(\omega+\omega_0)+C}_{\frac{1}{2}K(\omega+\omega_0)}}$$

$$H_{LP}(\omega) = \tfrac{1}{2}K(\omega) = \frac{A}{j\omega+C}$$

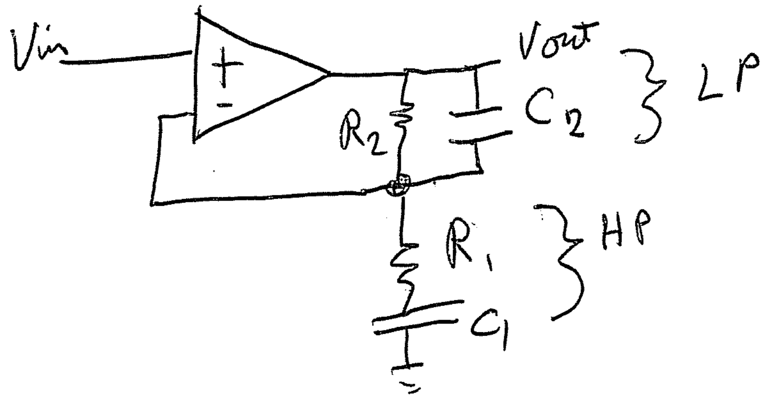
Active Filter Designs (i.e. contain an ideal op. amp).

Broad Band (no resonance) Low Q, Inverting

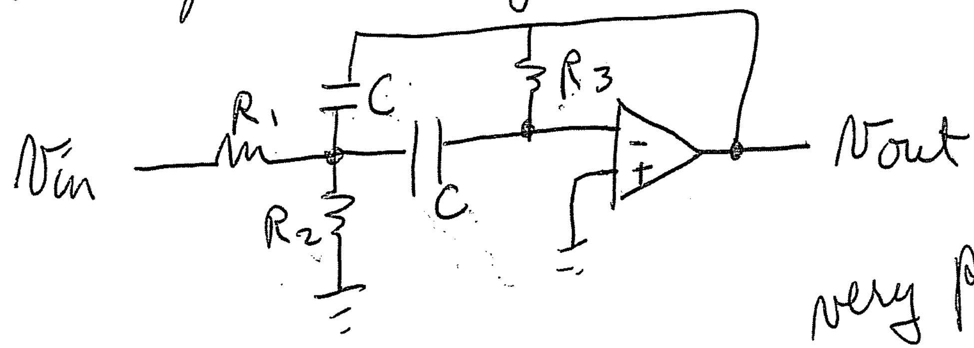


$$H(s) = -\frac{Z_f(s)}{Z_{in}(s)}$$

Non-inverting (~~[illegible]~~ Techniquely an allpass) Low Q, may not be practical

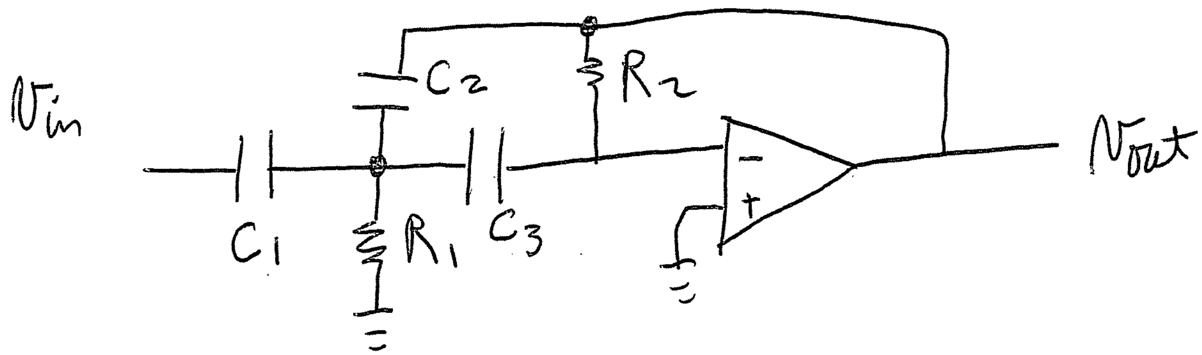


Bandpass (high Q)

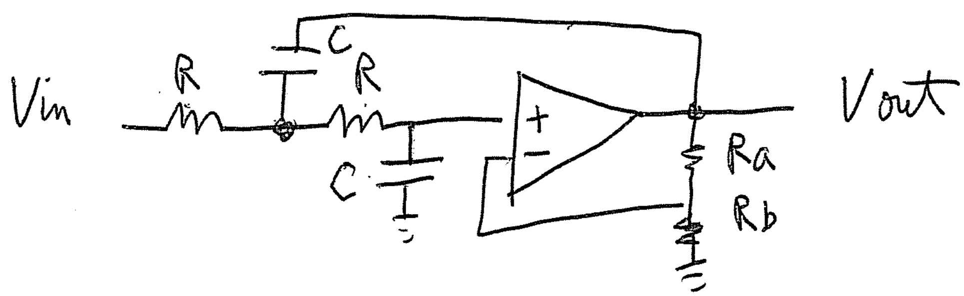


very practical

High Pass (High Q)



Low Pass (High Q)

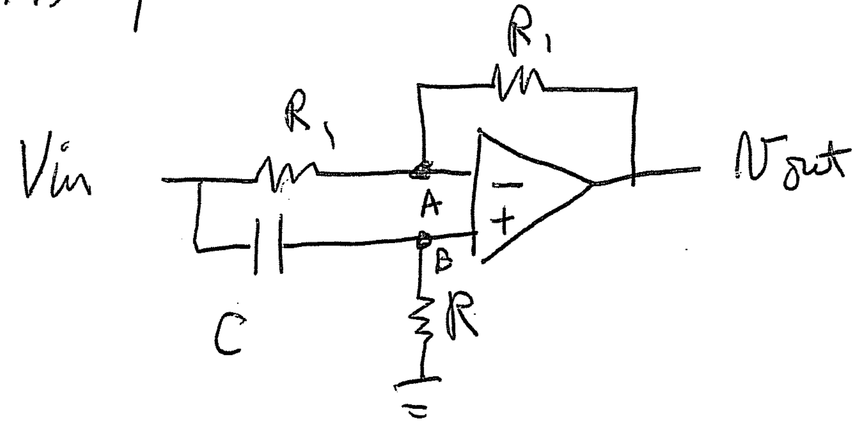


$$H(j\omega) = \frac{K\omega_0^2}{-\omega^2 + j\,\omega_0(3-K) + \omega_0^2}$$

where $K = \dfrac{R_a + R_b}{R_b}$

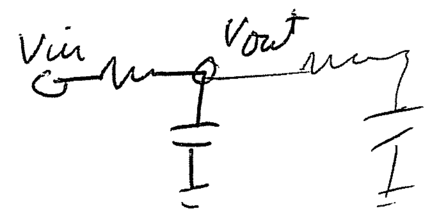
$$\omega_0 = \frac{1}{RC}$$

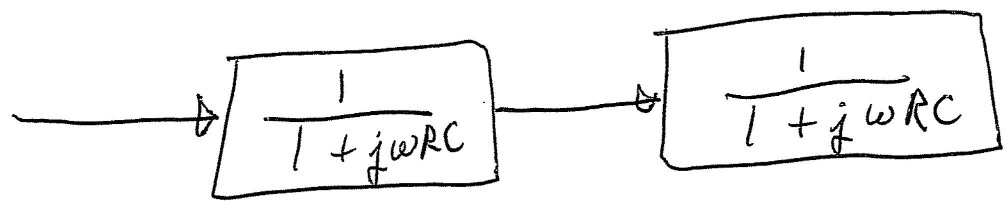
All pass Linear Phase Delay



Ex: 1 Consider cascaded 1st order LP filter

$$H(\omega) = \frac{1}{1 + j2\omega RC - (\omega RC)^2}$$





$$H(\omega) = \frac{-1/(RC)^2}{\omega^2 - 2j\omega/RC - \frac{1}{(RC)^2}} = \frac{1/(RC)^2}{s^2 + s/{RC/2} + 1/(RC)^2}$$

$$= \frac{\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

As $\quad s/\frac{RC}{2} = \frac{s\omega_0}{Q}$ and $\omega_0^2 = \frac{1}{(RC)^2}$

then $\quad Q = 1/2 \quad \omega_0 = \frac{1}{RC}$