Lecture 9B Linear Systems

$$x(t) \longrightarrow \boxed{h(t) \longleftrightarrow H(f)} \longrightarrow y(t)$$

$$y(t) = \int_{-\infty}^{\infty} x(\lambda)\, h(t-\lambda)\, d\lambda$$

convolution

$$Y(f) = X(f)\, H(f) \quad \text{where } H(f) = \frac{Y(f)}{X(f)} \equiv \text{transfer function of filter}$$

$h(t) \equiv$ impulse response of filter

let $x(t) = \delta(t)$

where $y(t) = \int_{-\infty}^{\infty} \delta(\lambda)\, h(t-\lambda)\, d\lambda = h(t)$

Power Transfer Function

Consider an energy signal (finite length) $y(t)$ as a response $y(t) = x(t) * h(t)$

The autocorrelation is

$$R_{yy}(\tau) = \int_{-\infty}^{\infty} y(\lambda)\, y^*(\lambda-\tau)\, d\lambda$$

The Power Spectral Density is

$$P_{yy}(f) = \mathcal{F}\{R_{yy}(\tau)\} = Y(f)\, Y^*(f) = |Y(f)|^2$$

$$P_{yy}(f) = X(f)\, H(f)\, X^*(f)\, H^*(f) = |X(f)|^2\, |H(f)|^2$$

$$= P_{xx}(f)\, |H(f)|^2$$

$$|H(f)|^2 = \frac{P_{yy}(f)}{P_{xx}(f)} \equiv \text{Power Transfer Function}$$

Finding Power Transfer of a Power Signal

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} y^2(t)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-\infty}^{\infty} y_T^2(t)\,dt$$

Using Parseval's theorem

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-\infty}^{\infty} |Y_T(f)|^2\,df$$

$$= \int_{-\infty}^{\infty} \underbrace{\lim_{T\to\infty} \frac{|Y_T(f)|^2}{T}}_{\text{PSD of } y \text{ or } P_{yy}(f)}\,df$$

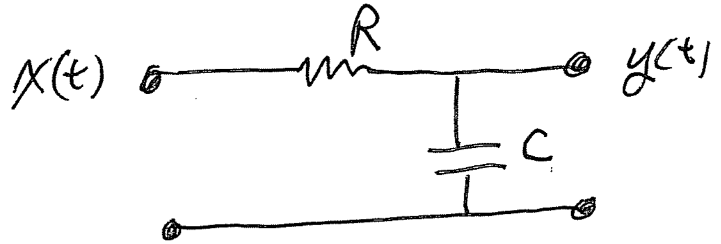
$$P_{yy}(f) = \lim_{T\to\infty} \frac{|Y_T(f)|^2}{T} \quad \text{and} \quad |Y_T(f)|^2 = |X_T(f)|^2\,|H(f)|^2$$

$$P_{yy}(f) = |H(f)|^2 \underbrace{\lim_{T\to\infty} \frac{|X_T(f)|^2}{T}}_{P_{xx}(f)}$$

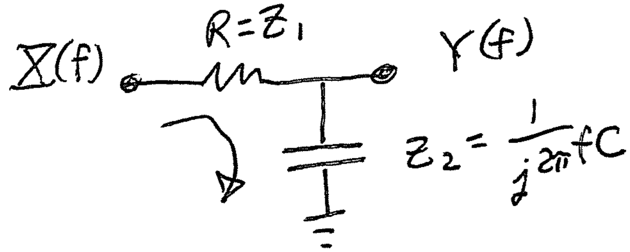
Power Transfer Function

$$\frac{P_{yy}(f)}{P_{xx}(f)} = |H(f)|^2$$

Ex: PTF of a simple circuit



$\Rightarrow$



$$H(f) = \frac{Y(f)}{X(f)} = \frac{Z_2}{Z_1 + Z_2} = \frac{\frac{1}{j2\pi fC}}{R + \frac{1}{j2\pi fC}} = \frac{1}{1 + j2\pi fRC}$$

Power Transfer function

$$|H(f)|^2 = \frac{1}{1+(2\pi f RC)^2}$$