Int J Adv Manuf Technol
DOI 10.1007/s00170-009-2465-1

ORIGINAL ARTICLE

Performance approximation of re-entrant lines with unreliable exponential machines and finite buffers

Yang Liu · Jingshan Li · Shu-Yin Chiang

Received: 10 July 2009 / Accepted: 20 November 2009

Abstract In this paper, we present an approximation method to estimate the production rate of re-entrant lines with unreliable exponential machines having identical processing times, finite buffers, and last buffer first serve scheduling policy. Recursive procedures are developed and structural properties are investigated. The results show that the proposed method provides an acceptable accuracy in production rate approximation for re-entrant lines.

Keywords Re-entrant lines · Production rate · Serial lines · Iteration procedures

1 Introduction

In many manufacturing systems, parts need to visit one or more machines multiple times. Such systems are referred to as re-entrant lines. For instance, multiple layers of material are imprinted on the wafer layer by layer in semiconductor manufacturing process. In automotive powertrain manufacturing plants, the ignition components are washed multiple times in order to keep them clean during production. Due to its wide applications in many manufacturing industries, in particular, in semiconductor and electronic manufacturing, the analysis, design, and management of re-entrant lines have significant importance.

Although a substantial amount of research effort has been devoted to throughput analysis of production systems, from serial lines to assembly lines and systems with complex operations, such as rework, parallel, split, merge and closed loops, etc. (see reviews [1–3] and monographs [4–7]), the study on performance analysis of lines with re-entrant operations is still limited. Most of the research effort on re-entrant lines focuses on studying the scheduling and control policies; less work is devoted to performance evaluation, in particular, of lines with unreliable machines and finite buffers. Queueing network model, fluid model, and mean value analysis have been the tools in re-entrant line research (see, for example, [8–13]). However, in these studies, infinite buffer capacity is typically assumed so that the blocking phenomenon is ignored. Stochastic petri-net provides another approach for analytical study of re-entrant lines (e.g., [14–17]). Analytical formulation for scheduling policy can be obtained using these methods. State space explosion can be one of the difficulties encountered in this method. The effort to overcome this difficulty has been made in recent studies [18]. Moreover, research has been carried out for systems with similar features to re-entrant lines, such as multiple part types, rework loops, etc. [19–25]. To extend the results to make them applicable to re-entrant lines, substantial effort is still required. In addition to analytical approaches, discrete event simulations have been used as a popular tool for re-entrant line studies to

Y. Liu · J. Li (✉)
Dept. of Electrical and Computer Engineering
and Center for Manufacturing, University of Kentucky,
Lexington, KY 40506, USA
e-mail: jingshan@engr.uky.edu

Y. Liu
e-mail: yang.liu@uky.edu

S.-Y. Chiang
Dept. of Information and Telecommunication
Engineering, Ming Chuan University, Taoyuan 333,
Taiwan, Republic of China
e-mail: sychiang@mail.mcu.edu.tw

Published online: 10 December 2009

evaluate performance, analyze scheduling and control policies, etc. (for example, [26–28]). Although detailed results can be obtained using simulations, long model development time and significant computation intensity limit its application.

Therefore, in spite of these advances, developing an efficient analytical method to estimate the performance of re-entrant lines is still needed [3, 29]. This paper is intended to contribute to this end. Specifically, as a starting point, we study the re-entrant lines with synchronous unreliable exponential machines and finite buffer capacities, i.e., all machines have identical cycle times and exponential up- and downtime. Lines with non-identical cycle times and non-exponential machines will be studied in future work. Since priority scheduling policy is typical in re-entrant lines, i.e., more processed jobs have higher priority than less processed ones, which is also referred to as last buffer first serve (LBFS) policy and has been shown as a stable policy [8], we focus our effort on re-entrant lines with LBFS scheduling policy.

The remaining part of the paper is structured as follows: Section 2 formulates the problem. The modeling and analysis method is presented in Section 3. Conclusions are given in Section 4.

2 Problem formulation

A typical structure of the re-entrant line under consideration is shown in Fig. 1, where the circles represent the machines and the rectangles are the buffers. The machines, the buffers, and their interactions are addressed by the following assumptions.

1) The system consists of M machines and $2M-1$ buffers separating two consecutive machines. The first time jobs are processed at machines m_i, $i = 1, \ldots, M$, and buffers b_{1i}, $i = 1, \ldots, M-1$. After first time processing at machine m_M, all jobs are sent to buffer b_{20}, waiting for second time processing. Then, the jobs are reprocessed at machines m_1 to m_M, but through buffers b_{2i}, $i = 1, \ldots, M-1$. Jobs leave the system after being processed at m_M for the second time.
2) All machines have identical processing times, normalized as one unit of time. The time is slotted as cycle time.
3) The up- and downtime of machine m_i are random variables exponentially distributed with parameters λ_i and μ_i, respectively. In other words, λ_i and μ_i are failure and repair rates, respectively, and $1/\lambda_i$ and $1/\mu_i$ are the average uptime and downtime, respectively.
4) Each buffer b_k, $k = 11, 12, \ldots, (1, M-1), 20, 21, 22, \ldots, (2, M-1)$, has capacity N_k, $0 < N_k < \infty$.
5) Machine m_i, $i = 1, \ldots, M-1$, is blocked by the first (respectively, second) time job at time t if it is up, buffer b_{1i} (respectively, b_{2i}) is full and machine m_{i+1} does not take part from it at time t. Machine m_M is blocked by the first time job at time t if it is up, buffer b_{20} is full, and machine m_1 does not take part from b_{20} at time t. Machine m_M is never blocked by the second time job.
6) The second time jobs have higher priorities than the first time ones. In other words, when it is up, machine m_i, $i = 2, \ldots, M-1$, always takes part from buffer $b_{2,i-1}$ if it is not empty and m_i is not blocked by b_{2i}; otherwise, it will take part from buffer $b_{1,i-1}$ if it is not empty and m_i is not blocked by b_{1i}. When machine m_1 is up and is not blocked by b_{21}, it takes part from buffer b_{20} if it is not empty; otherwise, new part will be loaded to be processed at m_1 if it is not blocked by b_{11}. Machine m_M will take part from $b_{2,M-1}$ if it is not empty; otherwise, m_M loads from $b_{1,M-1}$ if it is not empty and m_M is not blocked by b_{20}.
7) Machine m_i, $i = 2, \ldots, M$, is starved if it is up, both buffers $b_{1,i-1}$ and $b_{2,i-1}$ are empty, and their upstream machines fail to send any part into them. Machine m_1 is never starved by the first time job.

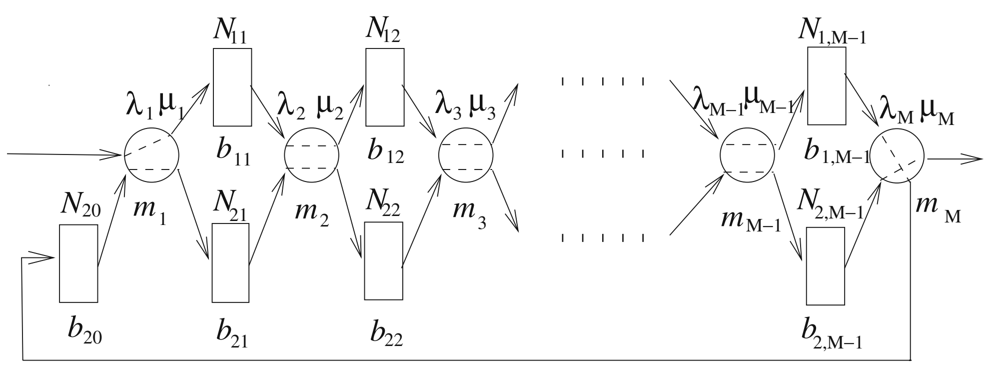


Fig. 1 Re-entrant lines

Let PR be the production rate of the system, i.e., the average number of finished parts (after second processing) produced by the last machine m_M per unit of time. The problem addressed in this work is formulated as follows:

> Given production system 1–7, develop a method to calculate the production rate as a function of the system parameters and investigate structural properties.

A solution to the problem is presented in Section 3.

3 Modeling and analysis

3.1 Idea of the approximation

The main difficulty of analyzing re-entrant line is that the machines are used for multiple processing of jobs. In addition to the complexity caused by blocking and starving, which is typically in serial lines, more difficulties, from the allocation of machine capacity to multiple processing of jobs, the priority loading and the dedicated dispatching policies, etc., make the exact analysis of system performance impossible. Therefore, approximation method is pursued in this work. Specifically, an iterative approach is developed to estimate the production rate of a re-entrant line with exponential machine reliability models. The idea of the method is to represent an M-machine re-entrant line in Fig. 1 by a $2M$-machine serial line (see Fig. 2 for illustration). The first M machines (denoted as m'_1 to m'_M) characterize the operations dedicated for the first time jobs and the second M machines (denoted as m''_1 to m''_M) for the second time jobs. Such an equivalence represents the path of the part flow within the system, i.e., they start visiting m_1 to m_M for first time processing, then, through buffer b_{20}, they return to m_1 to m_M for second time operations. The parameters of the machines (m'_i and m''_i, $i = 1, \dots, M$) are modified to take into account the shared processing of the first and second time jobs on them. Then, an iterative procedure is introduced to estimate these parameters recursively. When the procedure is convergent, an approximation of system production rate can be obtained.

To obtain the parameters of the modified machines, m'_i and m''_i, $i = 1, \dots, M$, an allocation of machine m_i to the first and second time jobs needs to be estimated. Since the second time jobs have higher priority than the first time ones, when the machine is up, m_i, $i = 1, \dots, M-1$, will load and process parts from buffer $b_{2,i-1}$ if it is not blocked by b_{2i} and starved by $b_{2,i-1}$. Machine m_M will carry out the last operation on jobs from $b_{2,M-1}$ if it is not empty and release the finished products when the work is completed. However, for the first time jobs, m_i, $i = 2, \dots, M$, will load from buffer $b_{1,i-1}$ only when the operations on second time jobs are not feasible (i.e., m_i is either starved or blocked by second time jobs). Similarly, a new job will be loaded by m_1 only when b_{20} is empty or m_1 is blocked by b_{21}. Therefore, it is equivalent to claiming that m_i is ready to process second time jobs as long as it is up, while it is allocated to first time jobs only when it is up and cannot work on second time jobs. Thus, machines m''_i can keep the same parameters as m_i; however, the parameters of m'_i need to be modified by enlarging their downtime. In other words, from the point of view of the first time jobs, the time when machine m_i is working on second time jobs is viewed as "downtime," since during such time, the machine is not available to the first time jobs.

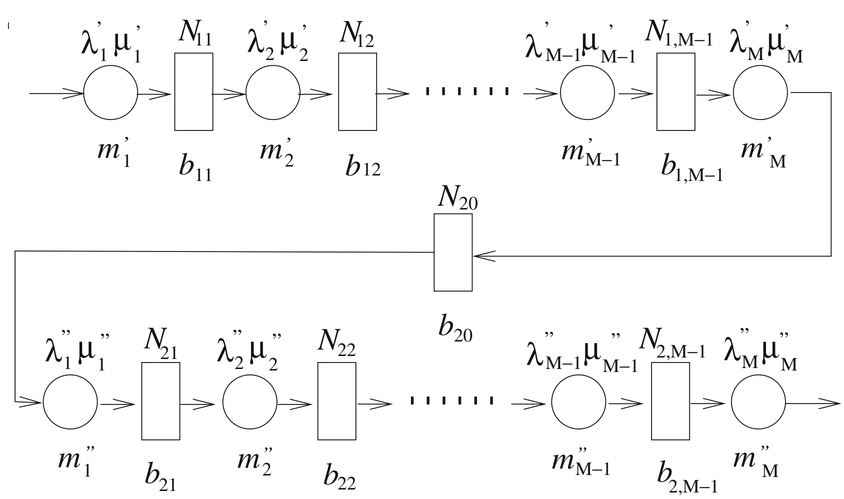


Fig. 2 Equivalent serial lines

Since, compared with uptime, machine downtime typically has larger impact on system performance, we modify downtime parameters, $\mu_i, i = 1, \ldots, M$, first.

$$\mu''_i = \mu_i, \quad i = 1, \ldots, M,$$
$$\mu'_i = \mu_i(\text{Prob}[b_{2,i-1} \text{ is empty}] + \text{Prob}[b_{2,i} \text{ is full}] - \text{Prob}[b_{2,i-1} \text{ is empty}]\text{Prob}[b_{2,i} \text{ is full}]), \quad i = 1, \ldots, M-1,$$
$$\mu'_M = \mu_i \text{Prob}[b_{2,M-1} \text{ is empty}].$$

The parameters of λ_i are modified so that the efficiency of the modified machine equals

$$e''_i = e_i = \frac{\mu_i}{\lambda_i + \mu_i}, \quad i = 1, \ldots, M,$$
$$e'_i = e_i(\text{Prob}[b_{2,i-1} \text{ is empty}] + \text{Prob}[b_{2,i} \text{ is full}] - \text{Prob}[b_{2,i-1} \text{ is empty}]\text{Prob}[b_{2,i} \text{ is full}]), \quad i = 1, \ldots, M-1,$$
$$e'_M = e_M \text{Prob}[b_{2,M-1} \text{ is empty}].$$

Therefore, we obtain

$$\lambda'_i = \lambda_i + \mu_i - \mu'_i, \quad \lambda''_i = \lambda_i, \quad i = 1, \ldots, M.$$

Since the probabilities of the buffer being empty or full are unknown, we introduce iterations. At the first step, assuming these probabilities are known, using a serial line analysis method, we analyze the $2M$-machine serial line with the modified parameters and obtain the probabilities that buffers $b_{2,i-1}$ and b_{2i}, $i = 1, \ldots, M$, are empty and full, respectively. Then, using these probabilities, we conduct the second iteration and continue until the procedure is convergent.

3.2 Recursive procedures

Clearly, in order to carry out the above procedure, a serial line analysis method is needed. Such a method has been developed in [7] and [20]. To make this paper self-contained, we introduce the method below:

Consider an M-machine serial production line with machine parameters $\lambda_i, \mu_i, i = 1, \ldots, M$, and buffer parameters $N_i, i = 1, \ldots, M-1$. In addition, all machines have identical cycle times, $c_i = 1$, $i = 1, \ldots, M$. Then, we have

Procedure 1

$$\begin{aligned}
\mu_i^b(l+1) &= \mu_i - \mu_i Q(\lambda_{i+1}^b(l+1), \mu_{i+1}^b(l+1), \lambda_i^f(l), \mu_i^f(l), N_i), i = 1, \cdots, M-1, \\
\lambda_i^b(l+1) &= \lambda_i + \mu_i - \mu_i^b(l+1), \\
\mu_i^f(l+1) &= \mu_i - \mu_i Q(\lambda_{i-1}^f(l+1), \mu_{i-1}^f(l+1), \lambda_i^b(l+1), \mu_i^b(l+1), N_{i-1}), i = 2, \cdots, M, \\
\lambda_i^f(l+1) &= \lambda_i + \mu_i - \mu_i^f(l+1),
\end{aligned} \tag{1}$$

with boundary conditions

$$\lambda_1^f(l) = \lambda_1, \quad \mu_1^f(l) = \mu_1, \quad \lambda_M^b(l) = \lambda_M, \quad \mu_M^b(l) = \mu_M, \quad l = 0, 1, 2, \ldots,$$

and initial conditions

$$\lambda_i^f(0) = \lambda_i, \quad \mu_i^f(0) = \mu_i, \quad i = 2, \cdots, M-1.$$

where l is iteration number, and

$$Q(\lambda_1, \mu_1, \lambda_2, \mu_2, N) = \begin{cases} \dfrac{(1-e_1)(1-\phi)}{1-\phi e^{-\beta N}}, & if \ \dfrac{\lambda_1}{\mu_1} \neq \dfrac{\lambda_2}{\mu_2}, \\[2ex] \dfrac{\lambda_1(\lambda_1+\lambda_2)(\mu_1+\mu_2)/(\lambda_1+\mu_1)}{(\lambda_1+\lambda_2)(\mu_1+\mu_2)+\lambda_2\mu_1(\lambda_1+\lambda_2+\mu_1+\mu_2)N}, & if \ \dfrac{\lambda_1}{\mu_1} = \dfrac{\lambda_2}{\mu_2}, \end{cases}$$

$$\phi = \frac{e_1(1-e_2)}{e_2(1-e_1)}, \tag{2}$$

$$\beta = \frac{(\lambda_1+\lambda_2+\mu_1+\mu_2)(\lambda_1\mu_2-\lambda_2\mu_1)}{(\lambda_1+\lambda_2)(\mu_1+\mu_2)},$$

$$e_i = \frac{\mu_i}{\lambda_i+\mu_i}, \quad i = 1, 2.$$

It is shown that the procedure is convergent so that

$$\lim_{s\to\infty} \mu_i^f(l) := \mu_i^f, \quad \lim_{s\to\infty} \mu_i^b(l) := \mu_i^b,$$
$$\lim_{s\to\infty} \lambda_i^f(l) := \lambda_i^f, \quad \lim_{s\to\infty} \lambda_i^b(l) := \lambda_i^b. \tag{3}$$

Then, the line production rate is obtained as

$$\mathrm{PR} = \frac{\mu_M^f}{\lambda_M^f + \mu_M^f} = \frac{\mu_1^b}{\lambda_1^b + \mu_1^b}. \tag{4}$$

Introduce operator

$$\Theta_{\mathrm{pr}}(\lambda_1, \mu_1, \ldots, \lambda_M, \mu_M, N_1, \ldots, N_{M-1}) \tag{5}$$

to denote the procedure calculating the production rate of such a serial line. Using this operator, let $\widehat{P}_{2i}^0$, $i = 1, \ldots, M$, be the probability that buffer $b_{2,i-1}$ is empty, and $\widehat{P}_{2i}^N$, $i = 1, \ldots, M-1$, the probability that buffer b_{2i} is full. In addition, let $\widehat{P}_{2M}^N$ represent the probability that b_{20} is full. Then, the following procedure can be developed for re-entrant lines:

Procedure 2

$$\mu_i'(s+1) = \mu_i(\widehat{P}_{2i}^N(s) + \widehat{P}_{2i}^0(s) - \widehat{P}_{2i}^N(s)\widehat{P}_{2i}^0(s)), \quad i = 1, \ldots, M,$$
$$\lambda_i'(s+1) = \lambda_i + \mu_i - \mu_i'(s+1), \quad i = 1, \ldots, M,$$
$$\widehat{P}_{2i}^N(s+1) = 1 - \Theta_{\mathrm{pr}}(\lambda_1'(s+1), \mu_1'(s+1), \ldots, \lambda_M'(s+1), \mu_M'(s+1), \lambda_1, \mu_1, \ldots, \lambda_M, \mu_M, N_{11}, \ldots, N_{1,M-1}, N_0, N_{21}, \ldots, N_{2,M-1}) / \Theta_{\mathrm{pr}}(\lambda_1'(s+1), \mu_1'(s+1), \ldots, \lambda_M'(s+1), \mu_M'(s+1), \lambda_1, \mu_1, \ldots, \lambda_i, \mu_i, N_{11}, \ldots, N_{1,M-1}, N_{20}, N_{21}, \ldots, N_{2,i-1}), \quad i = 1, \ldots, M-1,$$
$$\widehat{P}_{2M}^N(s+1) = 1 - \Theta_{\mathrm{pr}}(\lambda_1'(s+1), \mu_1'(s+1), \ldots, \lambda_M'(s+1), \mu_M'(s+1), \lambda_1, \mu_1, \ldots, \lambda_M, \mu_M, N_{11}, \ldots, N_{1,M-1}, N_0, N_{21}, \ldots, N_{2,M-1}) / \Theta_{\mathrm{pr}}(\lambda_1'(s+1), \mu_1'(s+1), \ldots, \lambda_M'(s+1), \mu_M'(s+1), N_{11}, \ldots, N_{1,M-1}),$$

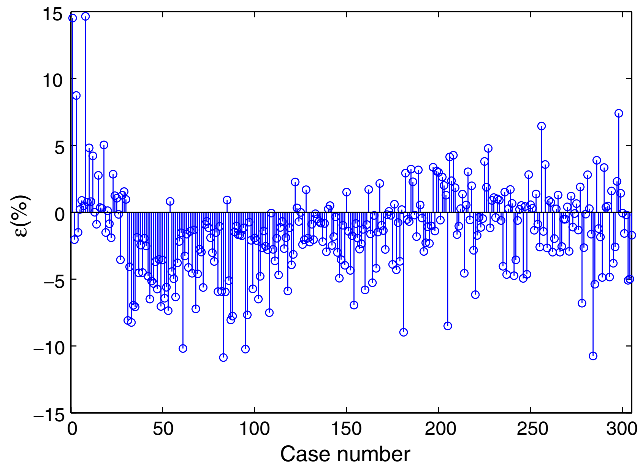


Fig. 3 Accuracy of Procedure 2

$$\widehat{P}_{2i}^0(s+1) = 1 - \Theta_{\mathrm{pr}}(\lambda_1'(s+1), \mu_1'(s+1), \ldots, \lambda_M'(s+1), \mu_M'(s+1), \lambda_1, \mu_1, \ldots, \lambda_M, \mu_M, N_{11}, \ldots, N_{1,M-1}, N_{20}, N_{21}, \ldots, N_{2,M-1}) / \Theta_{\mathrm{pr}}(\lambda_i, \mu_i, \ldots, \lambda_M, \mu_M, N_{2i}, \ldots, N_{2,M-1}), \quad i = 1, \ldots, M,$$
$$s = 0, 1, 2, \ldots,$$

with initial conditions

$$\widehat{P}_{2i}^N(0) \in (0,1), \quad \widehat{P}_{2i}^0(0) \in (0,1), \quad i = 1, \ldots, M,$$

and s is the iteration number.

Based on extensive numerical experiments, we discover that Procedure 2 returns two convergent sequences for even and odd iteration numbers. Thus, we formulate this as a numerical fact:

Numerical Fact 1 *Procedure 2 results in two convergent sequences, i.e.,*

$$\lim_{s\to\infty} \mu_i'(s) := \begin{cases} \mu_{i,\mathrm{even}}' & \text{if } s \text{ is even,} \\ \mu_{i,\mathrm{odd}}' & \text{if } s \text{ is odd,} \end{cases}$$
$$\lim_{s\to\infty} \lambda_i'(s) := \begin{cases} \lambda_{i,\mathrm{even}}' & \text{if } s \text{ is even,} \\ \lambda_{i,\mathrm{odd}}' & \text{if } s \text{ is odd.} \end{cases} \tag{6}$$

Table 1 Accuracy

Average $\lvert\epsilon\rvert$	max $\lvert\epsilon\rvert$	$\lvert\epsilon\rvert < 5\%$	$\lvert\epsilon\rvert < 10\%$
2.51%	14.65%	88.36%	97.82%

Introduce $\widehat{\mathrm{pr}}_{\mathrm{even}}$ and $\widehat{\mathrm{pr}}_{\mathrm{odd}}$ as the production rates with $\lambda'_{i,\mathrm{even}}$, $\mu'_{i,\mathrm{even}}$, and $\lambda'_{i,\mathrm{odd}}$, $\mu'_{i,\mathrm{odd}}$, respectively, we have

$$\begin{aligned}\widehat{\mathrm{pr}}_{\mathrm{even}} = \Theta_{\mathrm{pr}}(&\lambda'_{1,\mathrm{even}}, \mu'_{1,\mathrm{even}}, \ldots, \lambda'_{M,\mathrm{even}}, \mu'_{M,\mathrm{even}}, \lambda_1, \mu_1,\\ &\ldots, \lambda_M, \mu_M, N_{11}, \ldots, N_{1,M-1}, N_0, N_{21},\\ &\ldots, N_{2,M-1}),\\ \widehat{\mathrm{pr}}_{\mathrm{odd}} = \Theta_{\mathrm{pr}}(&\lambda'_{1,\mathrm{odd}}, \mu'_{1,\mathrm{odd}}, \ldots, \lambda'_{M,\mathrm{odd}}, \mu'_{M,\mathrm{odd}}, \lambda_1, \mu_1,\\ &\ldots, \lambda_M, \mu_M, N_{11}, \ldots, N_{1,M-1}, N_0, N_{21},\\ &\ldots, N_{2,M-1}).\end{aligned} \tag{7}$$

Then, the production rate of the re-entrant line is selected as the average of them, and denoted as $\widehat{\mathrm{PR}}$,

$$\widehat{\mathrm{PR}} = \frac{\widehat{\mathrm{pr}}_{\mathrm{even}} + \widehat{\mathrm{pr}}_{\mathrm{odd}}}{2}. \tag{8}$$

In this way, an estimate of the production rate of a re-entrant line in steady state is obtained.

3.3 Accuracy

The accuracy of the approximation is investigated numerically. Specifically, we consider more than 300 re-entrant lines by randomly and equiprobably selecting machine and buffer parameters from the following sets:

$$\begin{aligned} &M \in \{2, 3, 5, 10, 20, 50\},\\ &e_i \in [0.75, 0.95],\\ &T_{j,\mathrm{down}} = \frac{1}{\mu_j} \in [1, 20], \quad j = 1, \ldots, M,\\ &N_{ij} \in \lfloor k \cdot \max\{T_{j,\mathrm{down}}, T_{j+1,\mathrm{down}}\}\rfloor,\\ &\qquad i = 1, 2,\ j = 1, \ldots, M-1,\\ &N_{20} \in \lfloor k \cdot \max\{T_{M,\mathrm{down}}, T_{1,\mathrm{down}}\}\rfloor,\\ &k \in [1, 3],\end{aligned} \tag{9}$$

where $\lfloor x \rfloor$ denote the largest integer less than or equal to x. The machine average uptime, $T_{j,\mathrm{up}} = \frac{1}{\lambda_i}$, is

Table 2 Illustration examples

(a) $M = 2$							
λ_i	[0.0211, 0.1166]		μ_i	[0.0732, 0.3555]			
N_{11}	29	N_{21}	58	N_{20}	13		
PR	0.3481	$\widehat{\mathrm{PR}}$	0.3532	ϵ	1.46%		
(b) $M = 3$							
λ_i	[0.0049, 0.0391, 0.0048]						
μ_i	[0.0937, 0.1720, 0.1365]						
N_{1i}	[16, 23]	N_{2i}	[28, 15]	N_{20}	16		
PR	0.4030	$\widehat{\mathrm{PR}}$	0.3851	ϵ	−4.46%		
(c) $M = 5$							
λ_i	[0.0063, 0.0047, 0.0054, 0.0066, 0.0171]						
μ_i	[0.0759, 0.1774, 0.2674, 0.1174, 0.0652]						
N_{1i}	[26, 20, 6, 9]			N_{2i}	[28, 12, 8, 25]		
N_{20}	21	PR	0.3909	$\widehat{\mathrm{PR}}$	0.3832	ϵ	−1.95%
(d) $M = 10$							
λ_i	[0.0183, 0.0100, 0.0137, 0.0049, 0.0064, 0.0056, 0.0286, 0.0085, 0.0042, 0.0058]						
μ_i	[0.0551, 0.0569, 0.1373, 0.09554, 0.1001, 0.0610, 0.0745, 0.0984, 0.5841, 0.1663]						
N_{1i}	[51, 30, 34, 9, 26, 17, 45, 20, 23]						
N_{2i}	[52, 33, 9, 22, 18, 26, 30, 30, 2]						
N_{20}	4	PR	0.3401	$\widehat{\mathrm{PR}}$	0.3460	ϵ	1.71%
(e) $M = 20$							
λ_i	[0.0058, 0.0058, 0.1105, 0.0146, 0.0076, 0.0043, 0.0042 0.0147, 0.0070, 0.0049, 0.0044, 0.0185, 0.1977, 0.0060 0.0051, 0.0785, 0.0071, 0.0147, 0.0274, 0.0060]						
μ_i	[0.0710, 0.0973, 0.2639, 0.0744, 0.1638, 0.2261, 0.0536 0.0504, 0.0968, 0.4268, 0.2471, 0.0601, 0.0575, 0.0843 0.6829, 0.2379, 0.0847, 0.0954, 0.4501, 0.8722]						
N_{1i}	[37 18 27 4 18 12 8 19 54 16 4 6 48 46 36 2 9 19 16]						
N_{2i}	[37 17 6 38 11 12 48 58 31 5 5 39 19 17 2 9 31 16 5]						
N_{20}	3	PR	0.1144	$\widehat{\mathrm{PR}}$	0.1124	ϵ	−1.67%

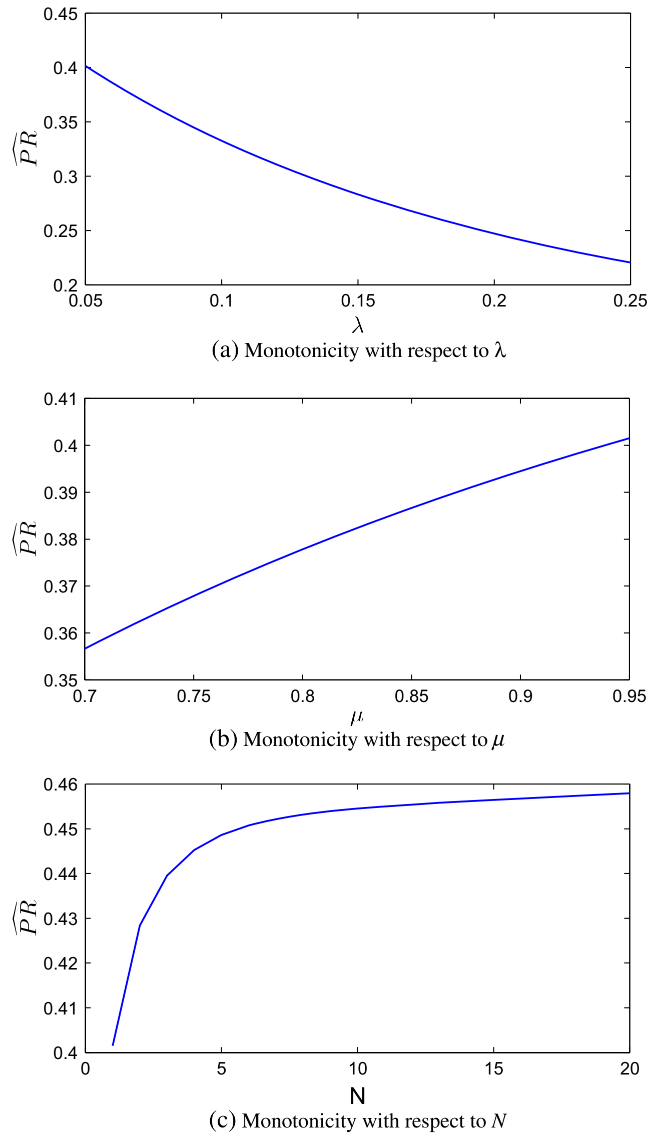
calculated through average downtime, $T_{j,\text{down}}$, and isolated efficiency, e_j, $j = 1, \ldots, M$.

For each of these lines, both analytical methods using Procedure 2 and simulation approach written in C++ code are pursued to evaluate system production rate. To ensure that the warm-up time is sufficient to guarantee that the steady state is reached, based on extensive experiments, 5,000 cycles of warm-up time are assumed in each simulation. Then, the data collection time (or run length) and number of replications are selected so that the effect of initial bias is minimal (by controlling the run length), and the variance of the estimates is also small (by controlling the number of replications), with reasonable computation efficiency. Again, based on experimental results, we select 200,000 cycles for collecting steady state statistics and 20 replications to obtain the average production rate. In this case, the 95% confidence intervals of the estimates are typically less than ±0.001. Then, the differences between analytical and simulation results are evaluated as

$$\epsilon = \frac{\widehat{\text{PR}} - \text{PR}}{\text{PR}} \cdot 100\%, \tag{10}$$

where PR and $\widehat{\text{PR}}$ are the production rates obtained by simulation and recursive procedures, respectively.

The results of this investigation are illustrated in Fig. 3. Table 1 provides the average and maximum errors in all the experiments and the percentages where errors are within 5% or 10%. It is shown that, in more than 88% of cases, the difference ϵ is within 5%, with a few exceptions (about 2% of cases) going up more than 10% (maximum to 14%). Thus, we conclude that, in all the cases we studied, Procedure 2 results in an acceptable accuracy for production rate estimation. For illustration purpose, five typical examples are provided in Table 2. From these results, we conclude that Procedure 2 provides a useful tool for design and analysis of re-entrant lines.



(a) Monotonicity with respect to λ

(b) Monotonicity with respect to μ

(c) Monotonicity with respect to N

Fig. 4 Monotonicity with respect to λ, μ, and N

3.4 Structural properties

It has been shown in the literature (e.g., [7]) that monotonicity holds in serial lines and assembly systems, i.e., system production rate can be improved by increasing machine reliability and/or buffer capacity. Similar properties are observed in re-entrant lines as well.

Monotonicity: The system production rate defined by Procedure 2 and estimate 8 is monotonically increasing with respect to μ_i, $i = 1, \ldots, M$, *and* N_{ij}, $ij = 11, \ldots, (1, M-1), 20, 21, \ldots, (2, M-1)$, *and decreasing with respect to* λ_i, $i = 1, \ldots, M$.

Illustrations of the monotonicity of PR with respect to λ_i, μ_i, and N_{ij} for a five-machine re-entrant line are shown in Fig. 4a, b, and c, respectively. In Fig. 4a, all machines are identical with failure rates λ_i ranging from 0.05 to 0.25, all repair rates μ_i are 0.95, and all buffers have capacity $N_{ij} = 1$. In Fig. 4b, machines are still identical but with all $\lambda_i = 0.0.05$ and all μ_i increasing from 0.7 to 0.95, and all buffer capacities are still kept at 1. In Fig. 4c, buffers are identical with N_{ij} changing from 1 to 20; all machines have $\lambda_i = 0.05$ and $\mu_i = 0.95$. Clearly, decreasing machine failures, improving machine repair rates, or increasing buffer capacity result in improvement in system production rate. However, the improvement is gradually diminishing when buffer

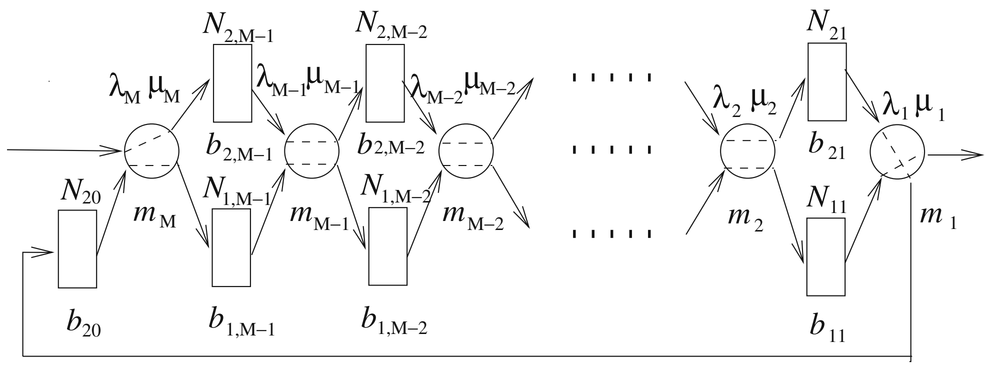


Fig. 5 Reversed re-entrant line

capacity is increased. Such results are similar to those discovered in serial line case.

It is clear that, for serial production lines, the production rate converges to the efficiency of the worst machine when buffers are infinite [7]. For re-entrant lines, since parts need to flow through each machine twice, and the production rates for first and second time processing jobs should be the same due to conservation of flow, we will obtain half of the worst machine efficiency as system production rate.

Asymptotic property: The system production rate defined by Procedure 2 *and estimate* 8 *satisfies*

$$\lim_{\substack{N_{ij}\to\infty \\ ij=11,\ldots,(1,M-1), \\ 20,21,\ldots,(2,M-1)}} \widehat{\mathrm{PR}} = \min_{i=1,\ldots,M} \frac{\mu_i}{2(\lambda_i+\mu_i)} = \min_{i=1,\ldots,M} \frac{e_i}{2}. \tag{11}$$

Reversibility is also observed in serial production lines [7]. For re-entrant lines, let the line in Fig. 5 be the reversed line of the one in Fig. 1. Higher priority is still assigned to buffer $b_{2i}, i = 1, \ldots, M-1$. Let $\widehat{\mathrm{PR}}$ and $\widehat{\mathrm{PR}}^{\mathrm{rev}}$ denote the production rates obtained through Procedure 2 and estimate 8, respectively. Then identical production rates are obtained.

Reversibility: The system production rate defined by Procedure 2 *and estimate* 8 *satisfies*

$$\widehat{\mathrm{PR}} = \widehat{\mathrm{PR}}^{\mathrm{rev}}. \tag{12}$$

The above properties have been verified through simulations.

4 Conclusions

Re-entrant lines are widely used in many manufacturing industries. In this paper, we present a method to approximate the system production rate of re-entrant lines with exponential machine reliability models. The numerical results suggest that this method can provide an acceptable precision for system production rate estimation. The future work will focus on extending the method to systems with multiple re-entrances, lines with asynchronous machines, non-identical machine parameters for re-entrant jobs, etc. The issues of multiple products, set up, work carriers, etc., are also to be investigated. The successful development of such methods will provide production engineers a quantitative tool for design and continuous improvement of re-entrant lines.

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