PRECISION SHAPE CONTROL OF NONLINEAR SHELLS AND PLATES

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ABSTRACT

Adaptive shape control is essential in many high-performance engineering systems, such as nozzles, airplane wings, helicopter blades, etc. Recent development of smart structures and structronic systems offers new alternatives to shape control with inherent and embedded actuator components. Imposed shape control often involves large deformations implying that the conventional linear theory is no longer applicable. This study is to explore a new structural control concept based on nonlinear theories. Nonlinear piezoelectric shell equations are derived based on von Karman geometric nonlinearity. Physical significance and application are discussed. As to compare the linear and nonlinear theories, a zero-curvature shell – plate is investigated. Analytical results suggest that the linear theory is indeed invalid when large deformation shape control is considered. Differences between the two theories are presented. Control effects of the plate with polymeric and ceramic piezoelectric actuators are compared.

1. INTRODUCTION

Adaptive shapes and surface control of structural and mechanical systems can revolutionize many engineering systems, such as flow control, precision manipulation and positioning, etc. Imposed shape change of shells and flat panels
of aerospace structures offers several aerodynamic advantages, such as lift control, flutter control, improved maneuverability, vibration and noise control, etc. Active shape control often involves large deformations and thus control of large deformation becomes an important issue in adaptive structures and structronic systems.

Geometrical nonlinearity of elastic shells were studied over the years (Librescu, 1987; Palazotto and Dennis, 1992; Pietraszkiewicz, 1979; Chia, 1980). A nonlinear model for a piezoelectric plate was recently proposed by Pat, et al (1993). Yu (1993) reviewed recent advances on modeling of linear and nonlinear elastic and piezoelectric plates. Linear thermo-electromechanical behavior and precision position control of distributed piezoelectric sensors and actuators were investigated (Tzou and Ye, 1994). Tzou and Howard (1993) also studied the thermo-electromechanical couplings of a single layer mm2 piezothermoelastic shell. However, geometrical nonlinearity of anisotropic piezothermoelastic shells simultaneously exposed to mechanical, electric, and thermal fields has not been fully investigated (Tzou and Bao, 1997; Tzou and Zhou, 1995).

This paper is concerned with a study of static and dynamic control of nonlinear flexible shells and plates subjected to mechanical, electric, and temperature excitations. A generic nonlinear thermo-piezoelectric shell is evaluated and its nonlinear thermo-electromechanical equations are derived based on the Hamilton's principle. Thermo-electromechanical couplings among the elastic, electric, and temperature fields are discussed, and nonlinear components identified. Application of the theory to control of wing box panels (plates) is demonstrated. Boundary control induced by the distributed piezoelectric actuator is investigated. Active control effects on nonlinear static deflections and natural frequencies imposed by the piezoelectric actuators are investigated.

2. NONLINEAR THERMO-PIEZOELECTRIC SHELL

Fundamental thermo-electromechanical properties of a piezoelectric continuum are briefly reviewed. Since shape control naturally involves large deformation geometric nonlinearity, a generic nonlinear thermo-piezoelectric shell based on the von Karman nonlinear theory is investigated and its governing equations are derived in this section. Thermo-electromechanical couplings are evaluated and its structural control applications are discussed. Figure 1 illustrates a generic double curvature piezoelectric shell continuum.

Fig.1 A double-curvature piezoelectric shell.

2.1 Thermopiezoelectric Constitutive Equations

Electric fields $\mathbf{E}_1, \mathbf{E}_2, \mathbf{E}_3$ and electric potential $\phi$ in a shell curvilinear shell coordinate system, Figure 1, are defined by (Tzou and Zhong, 1993)

$$
\begin{align*}
\begin{bmatrix}
\mathbf{E}_1 \\
\mathbf{E}_2 \\
\mathbf{E}_3
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & \frac{\partial}{\partial \alpha_1} \\
0 & f_{22} & 0 \\
0 & 0 & f_{33}
\end{bmatrix}
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\alpha_3
\end{bmatrix}
\end{align*}
\tag{1}
$$

where $f_{ij}(\alpha_1, \alpha_2, \alpha_3) = \Lambda_i(1+\alpha_3/R_i) \ (i=1,2)$. $f_{ij}(\alpha_1, \alpha_2, \alpha_3) = 1$; $\alpha_3$ is a finite distance measured from the reference surface; $\Lambda_1$ and $\Lambda_2$ are Lamé parameters. $R_1$ and $R_2$ are the radii of curvature of the $\alpha_1$ and $\alpha_2$ axes on the neutral surface defined by $\alpha_3 = 0$. Thermo-electromechanical constitutive relations of a generic piezoelectric shell continuum are governed by three equations: 1) the stress equation, 2) the electric displacement equation, and 3) the thermal entropy equation (Tzou and Bao, 1997).

$$
\begin{align*}
\{T\} &= \{c\} \{S\} - \{e\} \{E\} - (\lambda) \theta, \\
\{D\} &= \{e\} \{S\} + \{p\} \{E\} + \{p\} \theta, \\
\mathcal{S} &= (\lambda) \{S\} + \{p\} \{E\} + \alpha, \theta
\end{align*}
\tag{2}
$$

where $\{T\}, \{S\}, \{E\}$ and $\{D\}$ are respectively the stress, strain, electric field and electric displacement vectors; $\{c\}, \{e\}, \{p\}$ denote the elastic stiffness coefficient, piezoelectric coefficient and dielectric permittivity matrices, respectively; $\mathcal{S}$ is the thermal entropy density; $\theta$ is the temperature rise ($\theta = \Theta - \Theta_0$).
where \( \Theta \) is the absolute temperature and \( \Theta_0 \), the temperature of natural state in which stresses and strains are zero; \( \{ \lambda \} \) is the stress-temperature coefficient vector; \( \{ p \} \) is the piezoelectric coefficient vector; and \( \alpha_i \) is a material constant (\( \alpha_i = \rho c_i \Theta_0 \) where \( \rho \) is the material density and \( c_i \) is the specific heat at a constant volume). \( \{ ^t \} \) and \( \{ ^t \} \) are matrix and vector transpose, respectively. Nonlinear piezoelectric shells are evaluated and generic thermo-electromechanical equations and boundary conditions defined next.

### 2.2 Large Deformation Geometric Nonlinearity

As discussed previously, imposed shape control often involves large deformations—the geometric nonlinearity. A generic nonlinear deflection \( U_i \) in the \( i \)-th direction of the shell can be expressed as a summation of a (in-plane) membrane displacement \( u_i(\alpha_1, \alpha_2, t) \) and a higher order nonlinear shear deformation effect represented by the summation of angular rotations \( \beta_i(\alpha_1, \alpha_2, t) \):

\[
U_i(\alpha_1, \alpha_2, \alpha_3, t) = u_i(\alpha_1, \alpha_2, t) + \sum_{j=1}^{m} \alpha_j \beta_j(\alpha_1, \alpha_2, t), \quad i=1,2,3.
\]

(5)

where \( \beta_1 \) and \( \beta_2 \) represent the rotational angles in the positive sense of the \( \alpha_1 \) and \( \alpha_2 \) axes, respectively; and \( \beta_3 = 0 \). This expression includes higher order nonlinear shear deformation effects. However, according to the Love-Kirchhoff thin shell assumptions and a linear displacement approximation (first order shear deformation theory), only the first term is kept in the equation, i.e., \( m = 1 \) (Tzou, 1993). The displacements and rotational angles are independent variables in thick shells. However, the rotational angles are dependent variables in thin shells, and they can be derived from the thin shell assumptions in which the transverse strain normal \( S_1 \) is negligible and shear strains \( S_2 \) and \( S_3 \) are zeros (Soedel, 1993). Based on the thin shell assumptions, the rotational angles \( \beta_1 = \beta_1(\alpha_1) \) and \( \beta_2 = \beta_2(\alpha_2) \) are derived from the transverse shear strain equations, i.e., \( S_1 = 0 \) and \( S_2 = 0 \).

\[
\beta_i = \frac{u_i}{R_i} - \frac{1}{\alpha_{ij}} \frac{\partial u_j}{\partial \alpha_i}, \quad i = 1,2.
\]

(6)

In general, \( \alpha_i/R_i < < 1 \) \( \alpha_i/R_i < < 1 \), thus, the ratios of the finite distance to the radius of curvature are negligible, i.e., \( f_1 = 1 \) and \( f_2 = 1 \). Although it is assumed that the piezoelectric shell experiences large deformations in three axial directions, however, in reality, the in-plane deflections are still much smaller than the transverse deflections. Thus, the nonlinear effects due to the in-plane large deflections are usually neglected, i.e., the von Karman-type assumptions (Palazotto and Dennis, 1992; Chia, 1980). The nonlinear strain-displacement relations of a thin shell with a large transverse deflection \( u_i \) include a linear effect, denoted by a superscript \( \ell \), and a nonlinear effect, denoted by a superscript \( n \), induced by the large deformation:

\[
\begin{bmatrix}
S_1 \\
S_2 \\
S_6
\end{bmatrix} =
\begin{bmatrix}
\left[ \begin{array}{c}
\ell_1 \\
\ell_2 \\
\ell_6
\end{array} \right] +
\left[ \begin{array}{c}
n_1 \\
n_2 \\
n_6
\end{array} \right]
\end{bmatrix} + 
\begin{bmatrix}
k_1 \\
k_2 \\
k_6
\end{bmatrix}.
\]

(7)

where the subscripts 1 and 2 respectively denote two normal strains and 6 is the in-plane shear strain. Detailed membrane and bending strains are functions of displacements - \( u_i \)s.

#### Membrane strains:

\[
s_1 = \frac{1}{A_1} \frac{\partial u_1}{\partial \alpha_1} + \frac{u_2}{A_2} \frac{\partial A_1}{\partial \alpha_2} + \frac{u_3}{R_1} + \frac{1}{2} \left( \frac{\partial u_3}{\partial \alpha_1} \right)^2,
\]

(8)

\[
s_2 = \frac{1}{A_2} \frac{\partial u_2}{\partial \alpha_2} + \frac{u_1}{A_1} \frac{\partial A_2}{\partial \alpha_1} + \frac{u_3}{R_2} + \frac{1}{2} \left( \frac{\partial u_3}{\partial \alpha_2} \right)^2.
\]

(9)

\[
s_6 = \frac{1}{A_2} \frac{\partial u_1}{\partial \alpha_2} + \frac{1}{A_1} \frac{\partial u_2}{\partial \alpha_1} + \frac{u_1}{A_1 A_2} \frac{\partial A_1}{\partial \alpha_2} + \frac{u_2}{A_1 A_2} \frac{\partial A_2}{\partial \alpha_1} + \frac{1}{A_1 A_2} \frac{\partial u_3}{\partial \alpha_1} \frac{\partial u_3}{\partial \alpha_2}.
\]

(10)

#### Bending strains:

\[
k_1 = \frac{1}{A_1} \frac{\partial}{\partial \alpha_1} \left( \frac{u_1}{R_1} - \frac{1}{A_1} \frac{\partial u_1}{\partial \alpha_1} \right) + \frac{1}{A_1 A_2} \frac{\partial A_1}{\partial \alpha_2} \frac{\partial A_1}{\partial \alpha_2},
\]

(11)

\[
k_2 = \frac{1}{A_2} \frac{\partial}{\partial \alpha_2} \left( \frac{u_2}{R_2} - \frac{1}{A_2} \frac{\partial u_2}{\partial \alpha_2} \right) + \frac{1}{A_1 A_2} \frac{\partial A_2}{\partial \alpha_1} \frac{\partial A_2}{\partial \alpha_1},
\]

(12)

\[
k_6 = \frac{1}{A_2} \frac{\partial}{\partial \alpha_2} \left( \frac{u_1}{R_1} - \frac{1}{A_1} \frac{\partial u_1}{\partial \alpha_1} \right) + \frac{1}{A_1} \frac{\partial A_1}{\partial \alpha_2} \frac{\partial A_1}{\partial \alpha_2} - \frac{1}{A_1 A_2} \frac{\partial u_2}{\partial \alpha_2} \frac{\partial u_2}{\partial \alpha_2}.
\]

(13)

Note that the quadratic terms (nonlinear terms) inside the brackets are contributed by the large deflection. Membrane force resultants \( N_i \) and bending moments \( M_i \) of the thermo-piezoelectric shell laminate can be derived based on the induced strains (Tzou and Bao, 1997):
It is observed that there are three components, i.e., mechanical, electric, and temperature, in the force/moment expressions. Superscripts e and θ respectively denote the electric and temperature components. The membrane strains and bending strains are coupled by the coupling stiffness coefficients $B_{ij}$ in elastic force/moment resultantants. $N_{pi}$ and $N_{qj}$ are the electric and temperature induced forces; $M_{pe}$ and $M_{qθ}$ are the electric and temperature induced moments, respectively. In actuator applications, these electric forces and moments are used to control shell's static and dynamic characteristics. $A_{pi}$, $B_{pi}$, and $D_{pi}$ are the extensional, coupling, and bending stiffness constants. These strain/displacement relations, membrane force resultantants, bending moment resultantants, etc. are used in Hamilton's equation to derive thermo-electromechanical equations and boundary conditions of the nonlinear thermo-piezoelectric shells. Although it is assumed that the transverse shear deflections can be neglected, the integrated effect of the transverse shear stress resultantants $Q_{13}$ and $Q_{23}$ is neglected. Note that the effects of rotational inertias can be neglected in thin shells without large rotational effects.

2.3 Hamilton's Principle and Nonlinear System Equations

Hamilton's principle is used in deriving the shell thermo-electromechanical equations and boundary conditions of the piezothermoelectric shell continuum. Hamilton's principle assumes that the energy variations over an arbitrary period of time are zero. Considering all energies, one can write Hamilton's equation:

$$\delta \int_{t_1}^{t_2} \left( \frac{1}{2} \rho \dot{U}_j \dot{U}_j - \left( H(S, E, E, \Theta) + \Theta E \right) \right) dt - \int \left( (1, U_j, -Q_j, \varphi) dS \right) dt = 0. \quad (15)$$

where $\rho$ is the mass density; $H$ is the electric enthalpy; $t_1$ is the surface traction in the $\alpha_1$ direction; $Q_j$ is the surface electric charge; $\Phi$ is the electrical potential; $V$ and $S$ are the volume and surface of the piezothermoelectric shell continuum, respectively; and $U_j$ and $\dot{U}_j$ are the displacement and velocity vectors. It is assumed that only the transverse electric field $E_3$ is considered in the analysis. Substituting all energy expressions into Hamilton's equation and carrying out all variations, one can derive the nonlinear piezothermoelectric shell equations and boundary conditions of the piezothermoelectric shell continuum.

Note that all terms inside the bracess are contributed by the nonlinear effects. The nonlinear influence on the transverse displacement $u_3$ is significant. Also, the thermo-electromechanical equations are similar to standard shell equations. However, the force and moment expressions defined by mechanical, thermal, and electric effects are much more complicated than the conventional elastic expressions. Substituting the expressions of $N_{11}, N_{22}, N_{33}, M_{11}, M_{22}, M_{33}$ into the above equations leads to the thermo-electromechanical equations defined in the reference displacements $u_1, u_2, u_3$. The transverse shear deformation and rotatory inertia effects are not considered. The electric terms, forces and moments, can be used in controlling the mechanical and/or temperature induced excitations (Tzou and Ye, 1994).

Using Hamilton's equation, one can also derive all admissible mechanical and electric boundary conditions. Admissible mechanical boundary conditions on the boundary surfaces defined by a distance $\alpha_1$ and $\alpha_2$ are respectively summarized in Table 1 and Table 2.
Table 1 Boundary conditions (α₁ axis).

<table>
<thead>
<tr>
<th></th>
<th>Force/Moment</th>
<th>Displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>N₁₁ = N⁺₁₁</td>
<td>u₁ = u₁⁺</td>
</tr>
<tr>
<td>2</td>
<td>M₁₁ = M⁺₁₁</td>
<td>β₁ = β₁⁺</td>
</tr>
<tr>
<td>3</td>
<td>N₁₂ + M₁₂/(R₂) = V⁺₁₂</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Q₁₃ + (\frac{∂}{∂\alpha₂}(\frac{M₁₂}{A₂}))</td>
<td></td>
</tr>
</tbody>
</table>

Table 2 Boundary conditions (α₂ axis).

<table>
<thead>
<tr>
<th></th>
<th>Force/Moment</th>
<th>Displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>u₂ = u₂⁺</td>
</tr>
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<td>M₂₂ = M⁺₂₂</td>
<td>β₂ = β₂⁺</td>
</tr>
<tr>
<td>3</td>
<td>N₂₁ + M₂₁/(R₁) = V⁺₂₁</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Q₂₃ + (\frac{∂}{∂\alpha₁}(\frac{M₂₁}{A₁}))</td>
<td></td>
</tr>
</tbody>
</table>

The superscript * denotes the boundary forces, moments, displacements, and slopes. Usually, only either the force/moment boundary conditions or the displacement/slope boundary conditions are selected for a given physical boundary condition (Tzou, 1993; Soedel, 1993). In addition, additional transverse shear force terms Q₁ and Q₂ are nonlinear components induced by large deformations. These force terms do not appear in the linear case. The shear stress resultants are defined as

where V₁₃ and V₂₃ are the Kirchhoff effective shear stress resultants of the first kind; V₁₂ and V₂₁ are the Kirchhoff effective shear stress resultants of the second kind (Soedel, 1993). Note that all elastic, electric, and thermal related terms are included in the force and moment expressions. These electric terms can be used, in conjunction with control algorithms, as control forces/moments counteracting mechanical and temperature induced vibrations in distributed structural control of shells. Applications and simplifications of the nonlinear piezothermoelastic shell equations can be demonstrated in two ways: 1) material simplifications and 2) geometry simplifications. In the next section, the generic thermo-electromechanical equations are simplified to a zero-curvature shell - plate. Detailed comparisons of the linear and nonlinear theories are compared and numerical simulations are presented.

![Fig.2 - A nonlinear zero-curvature shell - plate with laminated distributed piezoelectric layers.](image)

3. CONTROL OF NONLINEAR PLATES

Plates are defined as zero-curvature shells, i.e., radii R₁ and R₂ are infinity and Lamé parameters A₁ and A₂ are 1's. Distributed piezoelectric
layers laminated (coupled or embedded) on elastic continua can be used as distributed sensors and/or actuators, Figure 2. Substituting the four parameters into the generic shell thermo-electromechanical equations yields the nonlinear plate equations

\[ -\frac{\partial(N_{xx})}{\partial x} + \frac{\partial(N_{yy})}{\partial y} + F_x = \rho h \dddot{u}_x, \] (23)

\[ -\frac{\partial(N_{yy})}{\partial y} + \frac{\partial(N_{yy})}{\partial x} + F_y = \rho h \dddot{u}_y, \] (24)

\[ \frac{\partial^2(M_{xx})}{\partial x^2} + \frac{\partial^2(M_{xx})}{\partial x \partial y} + \frac{\partial^2(M_{xy})}{\partial y^2} + \frac{\partial^2(M_{yy})}{\partial y^2} = \left[ N_{xx} \frac{\partial^2 u_z}{\partial x^2} + 2 N_{xy} \frac{\partial^2 u_z}{\partial x \partial y} + N_{yy} \frac{\partial^2 u_z}{\partial y^2} \right] + F_z = \rho h u_z, \] (25)

and forces and moments are defined by

\[ N_{ij} / M_{ij} = N_{ij}^m / M_{ij}^m + N_{ij}^e / M_{ij}^e + N_{ij}^c / M_{ij}^c, \] (26)

Injecting high voltages into the distributed piezoelectric actuators induces two major control actions. One is the in-plane membrane control force(s) and the other is the out-of-plane bending control moment(s). In general, the control moments are essential in planar structures, e.g., plates and beams; the membrane control forces are effective in shells. In this study, the piezoelectric actuators are used to control the nonlinear large deformation of flexible plates, and their control effectiveness is evaluated. Numerical solutions are derived to evaluate the control effectiveness of nonlinear plates in the case studies presented next.

4. CASE STUDY: PRECISION PLATE CONTROL

A simply supported nonlinear square plate (dimensions: 2ax2bxh) is considered in the case study. The piezoelectric laminated plate is subjected to both thermal and electric bending loads; \( \phi^1 = \phi^1 = \phi \) and \( \theta^1 = \Delta \theta \) where \( \Delta = \theta_1 - \theta_2 \) and \( \theta_1, \theta_2 \) are the temperatures on the top and the bottom of the plate, respectively. A nondimensional loading parameter \( \delta \) is defined by the piezoelectric constant, the moment arm, Young's modulus, the thermal stress coefficient, the plate thickness and width, and, of course, the temperature and the control voltage. The central deflections due to the nondimensional loading \( \delta \) calculated based on the linear and nonlinear theories are plotted in Figure 3. Center deflections of the simply supported nonlinear composite plate with control voltages and temperature loadings are analyzed and results are plotted in Figures 4-5. Note that the elastic plate is made of steels and the piezoelectric material layers are PZT materials. The plate dimensions are: steel thickness \( h_s = 1.0 \times 10^{-3} \), piezoelectric layer thickness \( h_p = 6.0 \times 10^{-3} \), plate length/width \( 2a = 0.5 \).
Fig. 5 - Center deflection of plate versus temperature.

Figure 2 shows the linear and nonlinear relationships between the plate deflection and the control voltage. This voltage-induced action can be used to counteract the deflections induced by the mechanical or temperature loadings, e.g., Figure 3. (Note that the PZT induced control action is superior to the polyvinylidene fluoride (PVDF) induced control action. This can be easily inferred from the inherent piezoelectric constants: $e_{31} = 10.43 \text{ C/m}^2$ for PZT while $e_{31} = 9.6\times10^{-3} \text{ C/m}^2$ for PVDF.) For comparison of control authorities between the PZT actuator and the PVDF actuator, analysis between deflection and control voltage of piezoelectric polymeric PVDF actuator is also conducted and plotted in Figure 6. Comparing Figures 4 and 6, one can clearly observe the superior control authority of the PZT actuator.

Figure 6. Center deflection versus control voltage (PVDF)

5. CONCLUSION

In the recent development of smart structures and structronic systems, piezoelectric materials are widely used as sensors and actuators in sensing, actuation, and control applications. This research is to investigate the control effectiveness of piezoelectric nonlinear flexible shells and plates (rectangular and square plates) subjected to mechanical and temperature excitations. It is assumed that the flexible shells and plates encounter the von Karman type geometrical nonlinearity. Nonlinear thermo-piezoelectric shell equations and generic boundary conditions were derived based on the von Karman geometric nonlinearity. Physical significance and application were discussed. As to compare the linear and nonlinear theories, a zero-curvature shell – plate was investigated.

Active control of nonlinear flexible deflections and thermal deformations using distributed piezoelectric actuators were studied, and their nonlinear effects were evaluated. Nonlinear static deflections with the influence of temperature and control voltage were studied. Voltage-temperature and displacement relations were investigated. The voltage imposed actuation can be used to compensate the nonlinear deformation and the temperature induced deformation. Simulation results also suggested that the voltage induced control displacement/force can be used to compensate the nonlinear static deflection, temperature effects, and natural frequencies of the piezoelectric laminated plate.

This study clearly revealed that the geometric nonlinearity theory has to be considered in the active structural and shape control of smart structures and structronic systems. Neglecting the nonlinear effect in the design and simulation studies can lead to catastrophic errors in reliability, safety, performance, etc.

6. ACKNOWLEDGEMENT

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7. REFERENCES


